المستوى الثالث - رياضيات - ميكانيكا تحليلية (٣٢٦ ر)

جامعة المنصورة **كلية العلوم** **قسم الرياضيات**	**المستوى الثالث** **شعبة رياضيات** **المادة: ميكانيكا تحليلية** **ر 326**	**الفصل الأول** **ديسمبر 2011** **الزمن: ساعتان** ٢٩/١٢/٢٠١١

أجب عن الأسئلة التالية:

1- أ) اذكر شروط تطبيق ميكانيكا لاجرانج على المنظومة الميكانيكية.

ب) اذكر شرط وجود صياغة لاجرانجية لمعادلات حركة منظومة معطى لها دالة هاملتون H.

ج) - خرزة تتحرك على سلك دائرى أملس مستواه رأسى بينما يدور السلك بسرعة زاوية ω حول قطره الرأسى، أوجد دالة لاجرانج وحل معادلة الحركة مستعينا بالتكامل الأول للحركة.

2- أ) عرف منظومة ليوفيل وبين كيفية حل مسألة حركتها بفصل المتغيرات .

ب) جسيم يتحرك فى المستوى تحت تأثير الجذب النيوتونى لمركزين ساكنين ثابتا جاوس لهما μ_1, μ_2 . بين أن المنظومة تأخذ شكل منظومة ليوفيل فى الإحداثيات الناقصية وبين كيفية حل مسألة الحركة بفصل المتغيرات .

3- أ) اذكر مبدأ هاملتون محددا شروط تطبيقه على المنظومة الميكانيكية

ب) أوجد الشكل الذى تأخذه كتينة ثقيلة موضوعة على سطح اسطوانى دائرى أملس محوره أفقى وطرفا الكتينة مثبتان فى نقطتين من السطح الذى تستند الكتينة بكاملها عليه.

4- أ) - أوجد المنحنيات فى النصف العلوى من المستوى xy التى تأخذ عليها الدالية التالية قيمها الصغرى: $S=\int_A^B \frac{\sqrt{dx^2+dy^2}}{y}$

ب) أوجد باستخدام مبدأ موبرتوى معادلة المسارات الممكنة لجسيم كتلته الوحدة فى المستوى يتحرك بطاقة E فى مجال جذب مركز نيوتونى جهده $V=-\frac{\mu}{r}$.

أستاذ المادة: أ. د./ حمد حلمى يحيى

المستوى الثالث - رياضيات، احصاء، علوم حاسب - نظرية القياس ر (٣١١)

Mansoura Univ.
Faculty of Science
Mathematics Dept.
Subject: Math.
Course Measure theory

3rdYear: math.
Date Jan.2012
Time: 2 hours
Full marks: 80

Answer the following questions:

[1] i) Define the measure function. What is the difference between measure function and probability function.
ii) Prove that the interval [0,1] is uncountable. [20 marks].

[2] i) Complete the following: A set S is measurable of for any set T.....
ii) Prove that if A,B are measurable sets then $A \cap B$ is measurable.
iii) Prove Poincare theorem. [20 marks].

[3] Prove that every bounded measurable function f on a set S is Lebesgue integrable on S. Hence or otherwise prove that Dirichlet function is Lebesgue integrable on [0,1]. Prove that it is not Riemann integrable on [01].
[20 marks].

[4] i) Evaluate the integral $\int_Q f$ where f is bounded measurable function and Q is the set of rational numbers.
ii) Prove that if f is measurable then its square f^2 is measurable.
[20 marks].

٣ رياضيات - تحليل مركب (ر٣١٢)

المستوى الثالث	كلية العلوم – قسم الرياضيات	الفصل الدراسى الأول
البرنامج: الرياضيات		الزمن: ساعتان
إسم المقرر: ر٣١٢ تحليل مركب		التاريخ: ٢٠١٢/١/٥

Answer the following questions:

1. a. Define entire function, smooth curve, multiple open contour.

   b. Find the roots of $(-4+4i)^{\frac{1}{4}}$.

   c. If $f(z)=u(x,y)+iv(x,y), z_0=x_0+iy_0, w_0=u_0+iv_0$. Then prove that $\lim\limits_{z\to z_0} f(z)=w_0$ if and only if $\lim\limits_{(x,y)\to(x_0,y_0)} u(x,y)=u_0$, $\lim\limits_{(x,y)\to(x_0,y_0)} v(x,y)=v_0$.

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2. a. State and prove the sufficient C.R.E's.

   b. Discuss the analyticity of $f(z)=z\operatorname{Re}(z)$.

   c. Prove that if $f(z)=u(x,y)+iv(x,y)$ is analytic in D. Then both u(x,y) and v(x,y) are harmonic in D.

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3. a. Show that $u(x,y)=x^3-3xy^2$ is harmonic function and find its harmonic conjugate v(x,y) so that $f=u+iv$ is analytic.

   b. Evaluate $\int\limits_{(0,3)}^{(2,4)} (2y+x^2)dx+(3x-y)dy$, $x=2t, y=t^2+3$.

   c. If $w=f(z)$ is analytic and $f'(z)$ is continuous. Then prove that $\int\limits_c f(z)dz=0$.

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4. a. Evaluate (i) $\int\limits_{c_+} \frac{z^2-\cos z}{(z+2i)^3}dz \; |z-1|=5$,

   (ii) $\int\limits_{c_+} \frac{\sin \pi z}{z^2+z-2}dz$, $c_+ : z=3e^{i\theta}, \theta\in[0,2\pi]$.

   b. Let f(z) be analytic in a simply connected domain D, $z_1, z_2 \in D$. Then prove that $\int\limits_{z_1}^{z_2} f(z)dz$ is independent of the path in D joining z_1 and z_2.

الممتحن : أ.د. محمد كمال عبد السلام عوف

المستوى الثالث - الرياضيات - تحليل عددى (٢١٢)
الاحصاء وعلوم الحاسب

3th Level Examination
Math 313
Numerical Analysis 1
Time : 2 hours
9/1/2012

ر ٣١٣
تحليل عددي ١
شعبة: الرياضيات + الأحصاء
وعلوم الحاسب

Mansoura University
Faculty of science
Department of mathematics

Answer the following questions (20 marks each)

Q1.

(i) Define the fixed point of a function g in [a,b]

(ii) Explain how to use the fixed point method to approximate the root of $f(x) = 0$ in [a,b], then state and prove the sufficient conditions to erasure the convergence of this method.

(iii) Prove that the Newton-Raphson method is quadratic convergent providing that $f'(p) \neq 0$.

Q2.

(i) Derive a suitable interpolation polynomial to approximation $f(1.1)$ from the following data

$f(x)$	1	1.5	2	2.5	3
x	2	3	5	4	10

(ii) Use an appropriate integration formula to approximate $\int_1^3 f(x)dx$ from the above date.

(iii) Find an approximate value of $f'(1.5)$.

Q3.

(i) Use Euler method to approximate the solution of the initial value problem

$$y' = t - y, \quad 1 \le t \le 2, \quad y(1) = 1.$$

at $t = 1.4$ (take $h = 0.2$)

(ii) Prove that

$$\int_{x_0}^{x_n} f(x)dx = \frac{h}{2}\left[f(x_0) + f(x_n) + \sum_{j=1}^{n-1} f(x_j)\right] - \frac{h^2}{12} f''(\xi)$$

(iii) Show that the sequence

$$P_n = \frac{2P_{n-1}}{3} + \frac{3}{P_{n-1}},$$

$P_0 > 0$ is convergent, then find the order of convergence.

Best Wishes.

Prof. E. M. Elabbasy

المستوى الثالث - رياضيات - جبر مجرد (٢) ، ٢١٥

Midterm Exam, 2012 **Time: 2 Hours** **Final Exam,** **12/1/2012**	**Faculty of Science - Math. Dept.**	**3rd year Math.,** **Subject: 315m** **Abstract Algebra (2)**

Answer the following questions: **Total (80 Marks)**

[1]-i) Define four only of the following : (20 Marks)

Integral domain, field, Quotient field of an integral domain, ideal, sylow p-subgroup.

ii) Prove that, the set of all units in Z_n forms a group under multiplication madulo n.

iii) the intersection of two subrings of aring is a subring. What about the union? Give an example.

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2) Find (five only) (20 Marks)

i) Asubring in the ring ZXZ.

ii) all solutions of the equation $x^3 - 2x^2 - 3x = 0$ in Z_6 .

iii) the characteristic of the rings : $Z_3 x Z_4$, ZxZ.

iv) all solutions of $45x \equiv 15 (\mathrm{mod}\, 24)$

v) the remainder of 3^{47} when it is divided by 23.

vi) all units in the rings : ZxQ, $Z[i], Z_{10}$.

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3) a) prove (two only) (20 Marks)

i) the set $S = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}; a, b \in Z \right\}$ is an ideal in the ring $(M_{2x2}(R);+;.)$

ii) Every finite integral domain is afield.

iii) Asylow p- subgroup of a finite group G is normal in G iff it is unique

b) Find the order of a sylow 3- subgroup of a group of order 54.

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4) Mark each of the following true or false. (20 Marks)

1- Agroup of order 27 is simple.

2- R is the field of quotients of the integral domain Z.

3- $M_{2x2}(Z_2)$ is an integraldomain

4) $Z_3 x Z_4$ is an abelian ring with unity

5) $R \cong C$ (as fieds).

6) the intersections of all ideals of aring is also an ideal of the ring.

7) $(2Z;+;) \cong (3Z;+;)$.

8) $a^{p-1} \equiv 1 (\mathrm{mod}\, p)$ for all integers a and prime p.

Good luck Dr. S. A.A

رابح جبر

المستوى الثالث - رياضيات - توبولوجي (١) (٣١٦)

Mansoura University	Third Year	First Term
Faculty of science	Mathematics	Date : 16/1/2012
Dept. of Math.	Topology (1)	Time :Tow hours

Answer the following questions **(20 Marks for each question)**

1- Let (X,τ) be a topological space and $A \subseteq X$. Prove that:

$(i) U \cap A = \phi, U \in \tau \Rightarrow U \cap A' = \phi$.

$(ii) A$ is open set $\Leftrightarrow b(A) \subseteq A'$.

$(iii) F$ is closed set $\Rightarrow \overline{F \cap A} \subseteq F \cap \overline{A}$.

2- (i) Prove that the property of being T_2- space is a topological property.

(ii) Prove that a topological space (X,τ) is normal space iff $\forall$ closed set F, $\forall$ open set U such that $F \subseteq U$ $\exists$ $V \in \tau$ such that $F \subseteq V \subseteq \overline{V} \subseteq U$.

3- (i) Let (X,τ) and (Y,δ) be two topological spaces. Prove that the mapping $f:(X,\tau) \to (Y,\delta)$ is continuous iff $\overline{(f^{-1}(B))} \subseteq f^{-1}\overline{(B)}, \forall B \subseteq Y$.

(ii) Prove that every closed subspace of normal space is normal.

4- (i) Prove that the compactness is a topological property.

(ii) Prove that the topological space (X,τ) is T_1-space iff $\{x\}$ is closed $\forall x \in X$.

المستوى الثالث - رياضيات ، احصاء وحاسب - نظرية الاحتمالات - (٣٣١ ر)

Mansoura University	**Final exam 1_st term**	**Subject. Prob.theory (1)**
Faculty of Science	**Time : 2 hours**	**Code : 331 math**
Math. Dept	**2011-2012**	**Date : 23/1/2012**
3_rd year	**Math.& Stat. and Computer Sci.**	**Total degree:80 mark**

Q1:(20 mark)

1- Let X and Y are two continuous random variables with a joint pdf of the form

$$f(x,y)=\begin{cases} k(x+y) & 0\le x\le y\le 1 \\ 0 & otherwise \end{cases}$$

Find each of the following

a) the constant k b) The marginals $f_1(x)$ and $f_2(y)$

c) The conditional pdf $f(^y/_x)$

2- Assume that X and Y are independent random variables with density functions

$$f(x)=\begin{cases} \frac{1}{2} & -1\le x\le 1 \\ 0 & otherwise \end{cases} \quad \text{and} \quad g(y)=\begin{cases} 1 & 0\le y\le 1 \\ 0 & otherwise \end{cases}.$$

Find the probability that the roots of the equation $h(t)=0$ are real, where $h(t)=t^2+2Xt+Y$.

Q2: :(20 mark)

Assume that X and Y are independent random variables with $X,Y\sim Geo(p)$.

a) Find the joint CDF, $F(x,y)$

b) If $T=X+Y$, find the joint CDF, $F(x,t)$

c) Find the marginal CDF's $F_X(x)$ and $F_T(t)$

Q3: (20 mark)

If $(X,Y)\sim MULT(n,p_1,p_2)$, then find

(i) Probability density function of Y.

(ii) $E(Y/x)$

(iii) The correlation coefficient ρ between X and Y.

Q4: (20 mark)

a) Let X is a random variable has the probability distribution

$f(x)=\frac{(x+2)}{15}$ $x=2,3,4$

Find the probability distribution of $Y=(X-3)^2$.

b) Let $X\sim N(0,1)$, find the pdf of the random variable $Y=X^2$.