٤ احصاء وعلوم حاسب - عمليات عشوائية (٤٣٢ر)

Mansoura university	**1st term**	ا لمقرر: عمليات عشوائية (١)
Faculty of science	**2011/2012**	الزمن : ساعتان
Math. Depart	**4th year**	التاريخ : ٢٠١١/١٢/٣١

Answer the following questions

Q1: (20 marks)

(a) Define the following

Stochastic process-Markov chain- Stationary Markov chain **(9 marks)**

(b) Let $\{N_1(t), t \geq 0\}$ and $\{N_2(t), t \geq 0\}$ be two Poisson processes having respectively rates λ_1, λ_2 and let $N(t) = N_1(t) + N_2(t)$, then show that $\{N(t), t \geq 0\}$ is a Poisson process with mean $(\lambda_1 + \lambda_2)t$. **(6 marks)**

(c) If $\{N(t),\ t \geq 0\}$ is a Poisson process and $s < t$, then for any k find

$$P\left[N(s) = k \big/ N(t) = n \right]$$ **(5 marks)**

Q2: (20 marks)

Let $\{X_n, n = 0,1,2,...\}$ is a Markov chain with state space $S = \{0,1,2\}$ has a transition probability matrix $P = \begin{bmatrix} 0.1 & 0.2 & 0.7 \\ 0.8 & 0.1 & 0.1 \\ 0.1 & 0.8 & 0.1 \end{bmatrix}$ and starting vector $\alpha_0 = (0.2, 0.5, 0.3)$. Find

(i) $P(X_3 = 2)$ (ii) $P(X_0 = 0, X_1 = 1, X_2 = 2)$ (iii) $P(X_5 = 2 | X_2 = 1)$

Q3: (20 marks)

Consider a Markov chain with state space $S = \{0,1,2\}$ and transition probability matrix $P = \begin{bmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{2} \end{bmatrix}$. Find the stationary distribution for it if exists.

Q4: (20 marks)

(a) Consider a Poisson process $\{N(t), t \geq 0\}$ with rate λ, and let T_n denote the elapsed time between the (n-1) _th and the n_th event. Find the probability distribution of T_n. **(10 marks)**

(b) Suppose that people immigrate into a territory at a Poisson rate $\lambda = 2$ per day

(i) What is the expected time until the eleventh immigrant arrives?

(ii) What is the probability that elapsed time between ninth and tenth arrival exceeds 3 days? **(10 marks)**

جامعة المنصورة – كلية العلوم
قسم الرياضيات
التاريخ : ٢١ - ١ - ٢٠١٢ م
الدرجة الكلية : ٨٠ درجة
الزمن : ساعتان

المستوى الرابع
احصاء حاسب
نظرية احصائية
(ر ٤٢١)

امتحان دور يناير ٢٠١٢
برنامج : إحصاء وعلوم الحاسب
المستوى: الرابع
اسم المقرر : نظرية إحصائية (٢)
كود المادة : ر٤٣١

أجب عن الأسئلة الآتية :-

السؤال الأول: أ) اشرح اختبار نسبة الاحتمال المتوالى (١٠ درجات)

ب) أخذت عينة عشوائية ذات حجم n من مجتمع طبيعى وسطه الحسابى يساوى صفر و تباينه θ حيث θ مقدار ثابت موجب مجهول . أوجد أفضل منطقة رفض ذات حجم α لاختبار فرض العدم $H_0: \ \theta=\theta_0$ مقابل الفرض البديل $H_1: \ \theta > \theta_0$ حيث θ_0 ثابت موجب . (١٥ درجة)

السؤال الثانى: أ) اشرح بالتفصيل خطوات اختبار فرض إحصائي حول الفرق بين وسطي مجتمعين سواء كان تباين المجتمعين معلوم أو غير معلوم. (١٠ درجات)

ب) إذا كان X متغير عشوائى يتبع التوزيع الأسى ببارامتر θ . قارن بين المنطقتين الحرجتين $C_2=\{x: \ x<0.0725\}$, $C_1=\{x: \ x>1\}$ للاختبارين المتنافسين T_1 , T_2 لاختبار فرض العدم $H_0: \ \theta=2$ ضد الفرض البديل $H_1: \ \theta<2$ (يمكن الاكتفاء بمشاهدة واحدة) (١٥ درجة)

السؤال الثالث: أ) اذكر الفرق بين الاختبارات البارامترية (المعلمية) و الاختبارات الغير بارامترية (اللامعلميه) ثم تكلم عن اختبار مربع كاي للاستقلال موضحا سبب أنه ذات طرف أيمن فقط . (١٥ درجة)

ب) أخذت عينة عشوائية مكونة من عشرة أشخاص و طبق عليهم برنامج معين لإنقاص الوزن لمدة شهر و سجلت أوزانهم بالكجم قبل و بعد البرنامج فكانت النتائج كالتالي :

60	95	42	82	90	62	57	77	82	62	الوزن قبل البرنامج
60	80	42	79	85	67	55	65	73	53	الوزن بعد البرنامج

بفرض أن الأوزان قبل و بعد تطبيق البرنامج تتبع التوزيع الطبيعى . هل يمكن القول أن هذا النظام قد أفاد فى إنقاص الوزن ؟ استخدم مستوى معنوية $\alpha=0.05$ (١٥ درجة)

$Z_{0.05}=1.645$, $Z_{0.025}=1.96$, $t_{(0.025,10)}=2.23$, $t_{(0.025,9)}=2.26$,
$t_{(0.05,9)}=1.83$, $t_{(0.05,10)}=1.81$

مع أطيب التمنيات بالتوفيق

د. محمد جاد

كورس ٤٢١ ـ تطبيقات ٢
رياضيات

Faculty of Sciences **Department of Mathematics** **Course: Linear programming** **Date: 24/12/ 2011** **Course code: (M421)**	Mansoura University	**Year: level 4 (Math./Stat.)** **Full mark: 80** **Time: 2 hours** **Semester: January**

Answer the following questions

Question 1-a: Mark true or false and correct the false ones

1. Any hyperplane divides $\mathbb{R}^n$ into two open half spaces.
2. A hyperplane is a closed and convex set.
3. Any continuous function on a compact set attains its minimum on the set.
4. The intersection of an infinite number of closed half spaces is called a polytope.
5. If $A = \{x_1, x_2\}$ then $H_{co}(A) = \{x: x = \alpha x_1 + (1-\alpha)x_2,\ 0 < \alpha < 1\}$.
6. A simplex in n-dimension has $n + 1$ vertices.
7. A matrix is called positive-semi definite if $z^t Az > 0$ for all $z \neq 0$.
8. The linear function $z = 2x_1 + 4x_2$ attains its extreme values at the vertices.
9. A feasible solution is a solution on which at least one of the constraints is violated.
10. The set of all feasible solutions forms a strictly convex set.
11. The simplex method solves any sort of linear programming problems.
12. A basic solution is a solution for the system of equations $Ax_B = b$.
13. Any basic solution is basic feasible solution.
14. Any feasible solution can be reduced to a basic feasible solution.
15. A convex combination of a finite number of different optimum solutions is also an optimum solution.
16. The simplex approach was developed in the late summer of 1947.
17. For only L.P.P., Min $z = -\text{Max}\ \{z\}$.
18. The dual of dual is primal. **[15 marks]**

b:- The strategic bomber command receives instruction to interrupt the enemy tank production. The enemy has four key plants located in separate cities, and destruction of any one plant will effectively halt the productions of tanks. There is an acute shortage of fuel, which limits the supply to 45000 litres for this particular mission. Any bomber sent to any particular city must have at least enough fuel for the round trip plus 100 litres. The number of bombers available to the commander and their descriptions are as follows:

Bomber type	Description	Km/ litre	Number available
A	Heavy	2	40
B	Medium	2.5	30

Information about the location of the plants and their probability of being attacked by a medium bomber and a heavy bomber is given below.

plant	Distance from base (km)	Probability of destruction by	
		A heavy bomber	medium bomber
1	400	0.10	0.08
2	450	0.20	0.16
3	500	0.15	0.12
4	600	0.25	0.20

Formulate it as a L.P.P. to see how many of each type of bombers should be dispatched, and how should they be allocated among the four targets in order to maximize the probability of success?

Please see overleaf **[5 marks]**

Question 2-a: Prove that a function $f: \Omega \to \mathbb{R}$ defined on a convex set $\Omega \in \mathbb{R}^n$ is convex if and only if for all $x, y \in \Omega, \lambda \in [0,1]$ then $f(\alpha x + (1-\alpha)y) \leq \alpha f(x) + (1-\alpha) f(y)$. Interpret the geometric meaning of convex function. **[7 marks]**

b:- Solve graphically the following L.P.P.: **Max** $z = x_1 + x_2$ subject to $-2x_1 + x_2 \leq 1, x_1 \leq 2$ and $x_1 + x_2 \leq 3, x_1, x_2 \geq 0$. **[5 marks]**

c:- State and prove the reduction of feasibility theorem. **[8 marks]**

Question 3-a: Use the big M-method to solve: **Max** $z = 3x_1 + 2x_2$ subject to $2x_1 + x_2 \leq 2$, $3x_1 + 4x_2 \geq 12, x_1, x_2 \geq 0$. **[10 marks]**

b:- Use the simplex method to solve:

$$\begin{array}{ll} \textbf{Max} & z = 2x_1 + 3x_2 \\ \textbf{Subject to} & -x_1 + 2x_2 \leq 4 \\ & x_1 + 3x_2 \leq 9 \\ & x_1 + x_2 \leq 6 \\ & x_1, x_2 \text{ Unrestricted} \end{array}$$

[10 marks]

Question 4: Given the objective function: $f(x) = 6x_1 + 7x_2 + 3x_3 + 5x_4$ and the inequality constraints,

$$\begin{gathered} 5x_1 + 6x_2 - 3x_3 + 4x_4 \geq 12, \\ x_2 + 5x_3 - 6x_4 \geq 10, \\ 2x_1 + 5x_2 + x_3 + x_4 \geq 8, \\ x_1, x_2, x_3, x_4 \geq 0 \end{gathered}$$

Find the optimum basis feasible solution that gives a minimum value for the objective function by using the dual simplex method. **[20 marks]**

Best wishes and good luck...
Dr Sameh Askar

المستوى الرابع - الرياضيات . نظرية الشبكات (٤١٨ ر)
احصاء وحاسب

El-Mansoura- Egypt	4^{th} Level .	المنصورة – مصر
Mansoura University	Program: Statistics and Computer Science	جامعة المنصورة
Faculty of Science	Subject: Lattice Theory	كلية العلوم
Mathematics Department	Course Code: Math. 418	قسم الرياضيات
First Term: Jan 2012	**Date: 14 Jan. 2012**	**Time: 2 hours**

<u>Answer the following five questions:</u>

1- Give two distinct equivalent definitions of lattices. (5 points)

And then give an example of each of: (each item 3 points)

a- A partially ordered set (poset) but not a lattice,

b- The smallest non- modular lattice,

c- A modular lattice but not distributive,

d- A distributive lattice having more than 4 elements,

e- An $\leq$ - homomorphism between two lattices but not $\vee$- homomorphism.

2- a- Let $(L, \wedge)$ be a meet semilattice as an algebra. (10 points)

Find a semilattice $(L, \leq)$ as a poset.

Give an example of a $\wedge$-semilattice but not $\vee$-semilattice.

b- Give all $\vee$-semilattices with 4-element set. (5 points)

c- Give an example of a modular lattice, but not distributive with 7 elements. (5 points)

3- For a commutative group $\boldsymbol{G} = (G ; \cdot)$.

a- Show that the set of all subgroups $\boldsymbol{S_N(G)}$ containing a subgroup N of $\boldsymbol{G}$ forms a lattice. (6 points)

b- Find the set of subgroups $\boldsymbol{S_{12Z}(\mathbb{Z})}$ of the group of integers $(\mathbb{Z}, +)$. (6 points).

c- Give the Hass Diagram of the lattice. $\boldsymbol{S_{12Z}(\mathbb{Z})}$. (8 points)

Is the lattice $\boldsymbol{S_{12Z}(\mathbb{Z})}$ distributive? . Why?

4- a- Let a , x, y be any three elements in a lattice $\boldsymbol{L} = (L ; \vee , \wedge)$. (10 points)

Prove that:

$$\boldsymbol{L} \text{ is modular } \Leftrightarrow \text{“ } a \wedge x = a \wedge y \;\&\; a \vee x = a \vee y \;\&\; x \leq y \Rightarrow x = y \text{ “.}$$

b- Give two equivalent definitions of a $\vee$-ideal of a lattice $\boldsymbol{L} = (L ; \vee , \wedge)$. (4 points)

c - Define a congruence relation θ on a lattice $\boldsymbol{L} = (L ; \vee , \wedge)$. (6 points)

And show that each congruence class $[a]\theta$ is a convex sublattice.

Examiner : Dr. Magdi H. Armanious **Full Mark: 80 points**

للمستوى الرابع - رياضيات - تحليل عددي (٢) (ر ٤١٢)
الإحصاء وعلوم الحاسب

4th Level Examination
Math 413
Numerical Analysis 2
Time : 2 hours
17/1/2012

ر ٤١٣
تحليل عددي ٢
شعبة: الرياضيات + الأحصاء
وعلوم الحاسب

Mansoura University
Faculty of science
Department of mathematics

Answer the following questions (20 marks each)

Q_1.

(i) Solve the system

$$2x_1 - x_2 + x_3 = -1,$$
$$3x_1 + 3x_2 + 9x_3 = 0,$$
$$3x_1 + 3x_2 + 5x_3 = 4,$$

using the LU-decomposition.

(ii) Find the linear least-squares polynomial to approximate the following data

x_i	2	4	6	8
y_i	2	11	28	40

(iii) Show that the boundary value problem

$$y'' = -\frac{2}{x}y' + \frac{2}{x^2}y + \frac{\sin(ln(x))}{x^2}, \quad 1 \le x \le 2,$$
$$y(1) = 1, \;\; y(2) = 2,$$

has a unique solution in $D = \{\, (x, y, y') \,|\, 1 \le x \le 2, \;\; -\infty < y < \infty, -\infty < y' < \infty \,\}$,

Then explain how to use the linear shooting method to approximate $y(1.1)$ [take $h = 0.1$]

Q_2.

(i) Define the spectral radius $\rho(A)$ of matrix A.

(ii) Apply Gauss-Siedel method to find the second approximation of the solution of the system

$$10x_1 - x_2 + 2x_3 = 6,$$
$$-x_1 + 11x_2 - x_3 + 3x_4 = 25,$$
$$2x_1 - x_2 + 10x_3 - x_4 = -11,$$
$$3x_2 - x_3 + 8x_4 = 15,$$

take $x^{(0)} = (0,0,0,0)^t$.

(iii) Prove that the sequence

$$x^k = Tx^{k-1} + c, \qquad k \geq 1$$

is convergent to the unique solution of $x = Tx + c$ iff $\rho(T) < 1$

(iv) Derive the SOR procedure to accelerate the convergence of Gauss-Siedel method.

Q3.

(i) Derive the Legender polynomials $\{P_n\}$ of degree 2 in $[-1,1]$ using Gram-Schmidt process. Then find the least-squares polynomial of degree 2 in $[-1,1]$ for the function $f(x) = e^{-x}$.

(ii) Let $T_n(x)$ denote the Chebyshev polynomial:

- show that $T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x), \; n \geq 1.$
- Show that $\{T_n\}$ are orthogonal in $[-1,1]$ with respect to $\omega(x) = \frac{1}{\sqrt{1-x^2}}$.

(iii) Find the trigonometric least-square polynomial of degree two for $f(x) = x$ on $[-\pi, \pi]$.

Best wishes

Prof. E. M. Elabbasy