المستوى الرابع - رياضيات - معادلات تفاضلية جزئية (٤٢٩)

Mansoura University
Faculty of Science
Mathematics Department
Final examination

Partial differential equation (Math429)
Fourth level Mathematics

Second term 2011-2012
Time Alowed Two Hours
Date 23-6-2012
Full marks 80

Answer the following questions:

Question One : **(20 marks)**

Prove that the solution of

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \; for -\infty < x < \infty$$

$$with \;\; Ics : u(x,0) = \varphi(x) \;\; and \;\; u_t(x,0) = \psi(x)$$

$$is :$$

$$u(x,t) = \frac{1}{2}[\varphi(x+ct) + \varphi(x-ct)] + \frac{1}{2c}\int_{x-ct}^{x+ct} \psi(s)ds$$

Question Two : (20 marks)

Solve

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},$$

$$BCs \; u(0,t) = 0 \;\; and \; \frac{\partial u}{\partial x}(1,t) = 0$$

Question Three (20 marks)

Solve $u_{tt} = c^2 u_{xx} - r u_t$, *with Dirichlet BC's:*

Question Four (20 marks)

a- Prove that if x=r cos θ, y =r sin θ, then

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$$

b- The Cartesian coordinates $(x, y) \in \mathbb{R}X\mathbb{R}$ and polar coordinates are related by

$x = r \cos\theta$, $y = r \sin\theta$.

Transform the equation $xu_y + yu_x = 0$ to the polar coordinates and then solve it (find a general solution)

المستوى الرابع. احصاء وحاسب رياضيات   نمذجة ومحاكاة ٤٢٦ ر

Mansoura University
Faculty of Science
Mathematics Department

Final exam
Second term
May 2012

4th level students (Mathematics / Statistics and computer Science programme)

Subject: Math 426 (Modelling and Simulations)

Date: 19 /06/2012 **Time allowed: Two hours**

Answer the following questions: **Total marks: 80**

Question one:

A) Define the following concepts: Logistic growth – predation – competition – mutualism. Give a discrete-time model for each of the first two concepts and a continuous-time model for each of the other two. **(10 marks)**

B) Consider the following Volterra model for fish:

$$\frac{dU}{dt} = \alpha U - pEU - \gamma UV, \qquad \frac{dV}{dt} = e\gamma UV - qEU - \beta V$$

where p and q are the catchability coefficients for the prey U and the predator V, respectively, and the fishing takes place with constant effort E. Explain the model, find the steady states and discuss their stability. **(10 marks)**

Question two:

A) Define the basic reproduction number. **(5 marks)**

B) Construct an SIR model for an infection spreading in a closed population with constant size and study the possibility to vaccinate a proportion p of the newborns and then find the critical vaccination coverage level required to eliminate the infection. **(15 marks)**

Question three:

A) Write down both the von Bertalanffy and Gompertz models for the tumour growth. **(5 marks)**

B) Prove that in a large **spherical** tumour there is a shell of proliferating cells, whose thickness depends on the excess nutrient concentration above a threshold $(c_2 - c_1)$, how fast the nutrient is consumed **k** and how fast it diffuses **D** according to the relation $h^2 = 2D(c_2 - c_1)/k$, but not on the size of the tumour itself. **(15 marks)**

Question four: **(20 marks)**

Assume that *p(a,t)* is the age-specific density of individuals aged *a* at time *t* in a demographically stationary population, $\mu(a)$ is the per capita age-dependent death rate as well as the per capita age-dependent birth rate and $B(t) = p(0,t) = \int_0^\infty \mu(a)p(a,t)da$ is the total number of births at time *t*. Write down the age-dependent model that describes the population dynamics and use the method of characteristics to find *p(a,t)*.

Best regards,
The examiner
Dr. Muntaser Safan

٤ رياضيات - ميكانيكا متقدمة (ر ٤٢٤)

جامعة المنصورة	المستوى الرابع	الفصل الثانى يونيو 2012
كلية العلوم	شعبة رياضيات	الزمن: ساعتان
قسم الرياضيات	المادة: ميكانيكا متقدمة	الدرجة الكلية 80

أجب على الأسئلة التالية:

1) أ- أثبت أن عزوم قصور جسم حول أى محاور متعامدة متقاطعة فى نقطة تحقق متباينات المثلث.

ب- عرف زوايا أويلر وعين بدلالتها متجه السرعة الزاوية منسوبا لمجموعة المحاور xyz **المثبتة فى الجسم.**

ج - استنتج تعبيرا عن كل من طاقة الحركة وكمية الحركة الزاوية للجسم المتماسك حول نقطة ثابتة منسوبة لمحاور ثابتة فى الجسم.

2) أ- اكتب معادلات الحركة لجسم متماسك مثبت من نقطة ويتحرك تحت تأثير وزنه.

ب- أوجد التكاملات الأولى العامة للحركة، مبينا علة وجود كل منها.

ج- أوجد التكامل الرابع فى حالة كوفاليفسكايا.

د - اذكر مع الإثبات تفسير بوانسو الهندسى لحركة الجسم المتماسك فى حالة أويلر.

3) بندول بسيط مكون من ثقل مثبت فى قضيب خفيف طوله l أزيح عن وضع اتزانه المستقر حتى أصبح أفقيا ثم ترك ليتحرك من سكون. عبر عن وضع الثقل بدلالة الزمن و أوجد الزمن الدورى للحركة.

4) أ- جسم متماسك له محور تماثل ومثبت من نقطة على هذا المحور ويتحرك تحت تأثير وزنه . اكتب معادلات الحركة وأوجد التكاملات الأولى للحركة.

ب- بين أن مسألة الحركة فى هذه الحالة يمكن حلها باستخدام الدوال الناقصية.

ج- إذا وضع الجسم بحيث يكون مركز كتلته رأسيا أعلى النقطة الثابتة فأوجد أقل سرعة زاوية تعطى له حول محوره بحيث يصبح وضع مركز الكتلة مستقرا.

درجات الأسئلة متساوية (20)

أستاذ المادة : أ. د. حمد حلمى يحيى

ع رياضيات - ١ النسبية العامة (٢٢٤ر)

Mansoura Univ.
Faculty of Science
Mathematics Dept.
Subject: Math.
Course General relativity 422

4thYear: math.
Date June 2012
Time: 2 hours
Full marks: 80

Answer the following questions:

[1] i) Why classical mechanics is not applicable to light. State special relativity principles. Derive Lorentz transformations. Derive length contraction, time dilation and apply them to the miu meson phenomena in cosmic rays. [20 marks].

[2] State Schwarzschild metric. Study bending of light ray. [20 marks].

[3] i) State cosmological principle. State Freedman-Robertson-Walker metric hence derive Hubble's law.
ii) Solve the cosmological models for the following cases: Static, and k=-1 cases. State their agreement with observations. $0=kc^2+R'^2+2RR''$. [20 marks].

[4] i) Write a short notice on Quasars. Can a quasar exist in our galaxy? justify your answer.
ii) If you see a blue star, what is its true color? Justify your answer.
iii) Can special relativity be applied on earth? Justify your answer. [20 marks].

المستوى الرابع - رياضيات - تحليل مركب / ٤١٧

الفرقة: المستوى الرابع	كلية العلوم	دور يونيه ٢٠١٢
الشعبة: رياضيات		الزمن: ساعتان
المادة: ر٤١٧ تحليل مركب(٢) (خ)		التاريخ: ٢٠١٢/٦/١٢

Answer the following questions:

1. a. Prove that if f(z) has a pole of order m at $z = z_0$. Then

$$\operatorname{Res}[f, z_0] = \lim_{z \to z_0} \frac{1}{(m-1)!} \frac{d^{(m-1)}}{dz^{(m-1)}} \left[(z - z_0)^m f(z)\right] \qquad \text{(10 marks)}$$

b. Evaluate $\displaystyle\int_{-\infty}^{\infty} \frac{\sin x}{x^2 - 2x + 2} dx$ (10 marks)

2. a. Define zero of order m for f(z). Prove that the zeros of an analytic function which is not identically zero are isolated. (5 marks)

b. Let f(z) be an analytic function inside and on C_+ (simple closed contour) except possibly for poles interior to C. Also let f have no zeros on C. Then prove that

$$\frac{1}{2\pi} A_c \arg f(z) = N - P,$$

where N and P are the number of zeros and poles of f(z). (10 marks)

c. Describe a Riemann surface for the multiple-valued function $w = z^{\frac{1}{4}}$. (5 marks)

3. a. Under the transformation $w = \sqrt{2} e^{i\frac{\pi}{4}} z + (2 - 3i)$, find the image of $R: x = y = 0$, $x = 2$ and $y = 1$ in the w-plane. (10 marks)

b. Prove that under $w = \frac{1}{z}$ straight lines and circles are mapped onto straight lines or circles. Find the image of $x + y - 2 = 0$ and $x - y + 2 = 0$ under $w = \frac{1}{z}$. (5 marks)

c. Define analytic continuation of f(z). (5 marks)

4. a. Let a function f(z) be analytic in a domain D that includes a segment of the x-axis and is symmetric to the x-axis. If f(z) is real whenever x is a point on that segment. Then prove that $f(\overline{z}) = \overline{f(z)}$, $z \in D$. Prove also if this condition is satisfied then f(z) is real. (10 marks)

b. Prove that $f(z) = \sin z$ is not bounded. Find the image of $x = c (c \neq \frac{\pi}{2} k$, k integer) under $w = \sin z$. (10 marks)

مع تمنياتنا بالنجاح والتوفيق

إسم الممتحن: أ.د./ محمد كمال عبدالسلام عوف

المستوى الرابع احصاء و حاسب - تحليل دالى ر ٤١٢
رياضيات

الشعب: ١. إحصاء و حاسب ٢. رياضيات. المادة: تحليل دالي- ر ٤١٤	كلية العلوم – قسم الرياضيات	دور مايو 2012 الزمن: ساعتان التاريخ: ٢/٦/2012

Answer the following questions:

[1] First: Objective Question: (20 marks)

Among the following statements mark the true and false ones with (√) and (×) respectively. Justify your answer for ONLY TWO of them :

(a) If $(\alpha_n), (\beta_n) \in \ell^4$, then $\sum_{n=1}^{\infty} |\alpha_n \beta_n| \le (\sum_{n=1}^{\infty} |\alpha_n|^4)^{1/4} . (\sum_{n=1}^{\infty} |\beta_n|^4)^{1/4}$ ....( )

(b) The sequence $\left(\frac{(-1)^n}{\sqrt[3]{n}} \right)$ belongs to ℓ^2. ....( )

(c) The dual space E^* of a normed space E is a Banach space. ....( )

(d) For a set $A = \{x_1, x_2, \ldots, x_n\} \subset \ell^\infty$, the linear hull H(A) is a separable space. ....( )

(e) Any two linearly isometric normed spaces are linearly homeomorphic. ....( )

(f) The space ℓ^4 is linearly homeomorphic to the space K^4. ....( )

(g) Any linear mapping $S : R \to R$ is bounded. ....( )

(h) A Banach space is a Hilbert space whose inner product is defined by a norm. ....( )

(i) Any inner product space can be converted to a metric space. ....( )

(j) If A is a closed subset of a Hilbert space H, then $A = (A^\perp)^\perp$. ....( )

Second: Subjective Questions (20 marks each)

[2] a. Let X be a metric space, $\Phi \ne A \subset X$ and $x \in X$. Prove that $x \in \overline{A}$ if and only if there is a sequence (a_n) in A with $\lim a_n = x$. [8 marks]

b. If T is linear transformation of a normed space E into another F, prove that the following statements are equivalent: [12 marks]

(i) T is continuous on E; ; (ii) T is continuous at the origin ; (iii) T is bounded .

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[3] a. Let $T: R^3 \to R^3$ be the mapping defined as follows : T(a,b,c) = (b+c , c+a , a+b) , for all $x = (a,b,c) \in R^3$. Show that $T \in L(R^3)$ and deduce that $\| T \| = 2$. [8 marks]

b. If E is a finite-dimensional normed space, show that :

(i) E is a Banach space ; [4 marks]

(ii) Any linear transformation $T: E \to F$ is bounded (F; being any normed space) [8 marks]

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[4] a. State and prove Schwarz' Inequality for inner product spaces. [10 marks]

b. Prove that if A is a closed subspace of a Hilbert space H, then

$$H = A \oplus A^\perp .$$ [10 marks]

Best wishes

El-Mansoura- Egypt	Final Exam. of 2^{nd} term 2012.	المنصورة - مصر	نظرية الأشكال
Mansoura University	Program: 4^{th} year.	جامعة المنصورة	٤ع /
Faculty of Science	Statistics and Computer Science	كلية العلوم	رياضيات
Mathematics Department	Subject: Graph Theory - Course Code: 412	قسم الرياضيات	احصاء وحاسب

Answer the following questions: **Marks**

1- Find each of:

(i) All simple graphs with 4 vertices. 4

(ii) A simple graph with 2 components , 10 vertices having: 4

(a) maximal number of edges. (b) minimal number of edges.

(iii) An example of a connected graph with $2n$ $(n \geq 2)$ vertices and no triangles and having: 4

(a) Maximal number of edges. (b) Minimal number of edges.

(iv) All nonisomorphic trees with 5 vertices. 4

(v) (a) Find the maximal number of arcs $|E(D)|$ of an oriented digraph D with n vertices . 4

(b) Find a regular graph G of order 1 with $2n$ vertices .

2- Prove.

(i) If there is an open walk between v_0 and v_n in a graph G, then there is a path between them. 5

(ii) If v is a vertex in the complete graph K_n , then $K_n - v$ is the complete graph K_{n-1}, and then show that K_n is not bipartite, $\forall\, n \geq 3$. 5

(iii) Let G be a connected graph with n vertices, then G or G^c has a triangle for each $n \geq 6$. 5

(iv) In any graph the number of vertices of odd degrees is even. 5

3- (i) Give the definition of a maximal planar graph. Let G = (V, E) be a maximal planar graph with $|V| = n$ and $|E| = m$. Prove that $m = 3n - 6$. and then show that K_5 is not planar. 10

(ii) Give the definition of the rooted tree $T(v_0)$ and prove that indeg $v = 0$ or 1 for each vertex v of T. And then use the rooted binary tree to sort and read the following list of words: 10

"Please help me alphabetize the following list of words".

4- (i) Let $T = (V, E)$ be a tree with n vertices and u, v be two non-adjacent vertices. Prove that $T + uv$ has precisely one cycle C. If e is an edge of C, then the graph $T + uv - e$ is once again a tree. 7

(ii) Prove that a graph G is regular of degree 2 $\Leftrightarrow$ each component of G is a cycle. Give an example of regular graph of degree 2 with two components. 7

(iii) Define the adjacency matrix A of a digraph D. Prove that the entry b_{ij} of A^2 represents the number of all diwalks of length 2 from the vertex v_i to the vertex v_j. 6

Total = 80

Good luck.
Dr. M. H. Armanious

ع رياضيات

هيدروديناميكا

Mansoura University
Faculty of Science
Course: OR

30-6-2012

Undergraduate (4^{nd} year exam)

Time:2Hours

Answer the following questions

No. of Questions:4

Total Mark:75

Question:1 (20 marks)

(a) **If it is possible solve the following mathematical models by using the graphical method.**

(i)
$$\begin{aligned} &\text{Minimiz} && Z = 2x_1 + 3x_2 \\ &\text{subject to} && x_1 + x_2 \geq 5 \\ & && 2x_1 + 4x_2 \leq 8 \\ & && x_1, x_2 \geq 0 \end{aligned}$$

(ii)
$$\begin{aligned} &\text{Minimiz} && Z = 2x_1 + 3x_2 \\ &\text{subject to} && -x_1 + x_2 \geq 5 \\ & && x_1 + x_2 \leq 8 \\ & && x_1, x_2 \geq 0 \end{aligned}$$

(iii)
$$\begin{aligned} &\text{Maximize} && Z = -x_1 + 3x_2 \\ &\text{subject to} && -x_1 + 3x_2 \leq 9 \\ & && x_1 + x_2 \leq 6 \\ & && x_1 - x_2 \leq 2 \\ & && x_1, x_2 \geq 0 \end{aligned}$$

(b) **Express the following *L.P* in the standard form:**

$$\begin{aligned} &\text{Maximize} && Z = 4x_1 + 2x_2 + 6x_3, \\ &\text{subject to} && 2x_1 + 3x_2 + 2x_3 \geq 6 \\ & && 3x_1 + 4x_2 = 6 \\ & && 6x_1 - 4x_2 + x_3 \leq 10 \\ & && x_1, x_2, x_3 \geq 0 \end{aligned}$$

Question:2 (20 marks)

(a) **Use *The big M-method* to solve**

$$\begin{aligned} &\text{Maximize} && Z = 3x_1 + 8x_2 \\ &\text{subject to} && x_1 + 3x_2 = 20 \\ & && x_1 \leq 8 \\ & && x_2 \geq 6 \\ & && x_1, x_2 \geq 0 \end{aligned}$$

(b) **Construct the dual to the primal problem**

$$\begin{aligned} &\textit{Maximize} && Z = 3x_1 + 10x_2 + 2x_3, \\ &\textit{subject to} && 2x_1 + 3x_2 + 2x_3 \leq 7 \\ & && 3\,x_1 - 2x_2 + 4x_3 = 6 \\ & && 6\,x_1 - 4x_2 + \ x_3 \geq 10 \\ & && x_1, x_2, x_3 \geq \end{aligned}$$

<u>(20 marks)</u>

<u>*Question:3*</u>

Find the initial feasible solution to the following transportation problem by:

(i) *north-west corner rule,*
(ii) *Minimum cost rule,*
(iii) *Vogel's approximation method*

To

		1	*2*	*3*	*4*	*Supply*
	1	*2*	*3*	*11*	*7*	*6*
From	*2*	*1*	*0*	*6*	*1*	*1*
	3	*5*	*8*	*15*	*9*	*10*
	Demand	*7*	*5*	*3*	*2*	

<u>*(15 marks)*</u>

<u>*Question:4*</u>

Solve the following *assignment* problem:

	I	II	III	IV	V
1	10	5	9	18	11
2	13	9	6	12	14
3	3	2	4	4	5
4	18	9	12	17	15
5	11	6	14	19	10

<u>***WITH THE BEST WISHES***</u>