المستوى الرابع - رياضيات
نظرية المعادلات التفاضلية (٤١١ر)

Faculty of science
Math-department

theory of differential equations
B.sc. Exam

January : 2013
Time : 2 H

Answer the following questions :

1-a) Give the solution $\begin{cases} \frac{dx}{dt} = A e^{\lambda x} \\ \frac{dy}{dt} = B e^{\lambda x} \end{cases}$ **and the path of the system** $\begin{cases} \frac{dx}{dt} = 3x - y, \\ \frac{dy}{dt} = 4x - y \end{cases}$ (10marks)

b) Define the following : Lipschitz condition , all types and stability of the critical points for any linear system , fundamental matrix and the orthonormal functions . (10marks)

2) State and prove the existence and uniqueness theorem of solution of the I.V **problem ,**
$\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$ on $R := \{(x, y) / |x| \leq a , |y| \leq b\}$. (20marks)

3-a) Discuss the existence and uniqueness solution of the D.E :

$$(x^2 - 4x + 5)\, y'' + \frac{1}{x-2} y' + y = 0, \quad y(0) = 3 \ , \ y'(0) = 0 \ .$$ (8marks)

b)If the vector functions $\varphi_1, \varphi_2, \ldots, \varphi_n$ **defined by** $\varphi_1 = \begin{pmatrix} \varphi_{11} \\ \varphi_{21} \\ \vdots \\ \varphi_{n1} \end{pmatrix}$. $\varphi_2 = \begin{pmatrix} \varphi_{12} \\ \varphi_{22} \\ \vdots \\ \varphi_{n2} \end{pmatrix}$,...., $\varphi_n = \begin{pmatrix} \varphi_{1n} \\ \varphi_{2n} \\ \vdots \\ \varphi_{nn} \end{pmatrix}$

are linearly dependent solutions of the H.L.V.D.E $\frac{dX}{dt} = A(t)\, X$, $y(x_0) = y_0$ **on** $I = [a,b]$.

Prove that the Wronskian $W(\varphi_1, \varphi_2, \ldots, \varphi_n)(t) = 0$ **(12marks)**

4-a) Discuss all types and stability of the critical points of the system $\begin{cases} \frac{dx}{dt} = 2x - 7y \\ \frac{dy}{dt} = 3x - 8y \end{cases}$, (8marks)

b) Give the orthonormal functions of the boundary value problem ,

$$\frac{d}{dx}\left[x \frac{dy}{dx}\right] + \frac{\lambda}{x} y = 0 \ , \ y'(1) = 0 , \ y'(e^{2\pi}) = 0 \ , \ \lambda \geq 0.$$ (12marks)

المستوى الرابع. احصاء رياضي
رياضيات / بحوث عمليات - ر٤٢١

Mansoura University
Faculty of science
Mathematics Department

29-12-2012
Course: OR   Code:R421
(4^{th} level exam)   Time:2Hours
Answer the following questions   No. of Questions:4   Total Mark:80

Question:1 (20 marks)

(a) If it is possible solve the following mathematical models by using the graphical method.

(i)
$$\begin{aligned} \text{Maximize} \quad & Z = 4x_1 + 5x_2, \\ \text{subject to} \quad & x_1 + x_2 \geq 5, \\ & 2x_1 + 4x_2 \leq 16, \\ & x_1, x_2 \geq 0. \end{aligned}$$

(ii)
$$\begin{aligned} \text{Minimize} \quad & Z = 4x_1 + 3x_2, \\ \text{subject to} \quad & -x_1 + x_2 \geq 3, \\ & x_1 + x_2 \leq 8, \\ & x_1, x_2 \geq 0. \end{aligned}$$

(iii)
$$\begin{aligned} \text{Maximize} \quad & Z = x_1 + 3x_2, \\ \text{subject to} \quad & -x_1 + 3x_2 \leq 9, \\ & x_1 + x_2 \leq 6, \\ & x_1 - x_2 \geq 2, \\ & x_1, x_2 \geq 0. \end{aligned}$$

(b) Express the following ***L.P*** in the standard (matrix) form:

$$\begin{aligned} \text{Maximize} \quad & Z = 4x_1 + 2x_2 + 6x_3, \\ \text{subject to} \quad & 2x_1 + 3x_2 + 2x_3 \geq 5, \\ & 3x_1 + 4x_2 = 8, \\ & 6x_1 - 4x_2 + x_3 \leq 10, \\ & x_1, x_2, x_3 \geq 0. \end{aligned}$$

Question:2 (20 marks)

(a) Use ***The big M-method*** to solve

$$\begin{aligned} \text{Maximize} \quad & Z = 3x_1 + 2x_2, \\ \text{subject to} \quad & 2x_1 + x_2 \leq 1, \\ & 3x_1 + 4x_2 \geq 4, \\ & x_1, x_2 \geq 0. \end{aligned}$$

(b) Construct the dual to the primal problem:

$$\begin{aligned}
\textit{Maximize} \quad & Z = 3x_1 + 10x_2 + 2x_3, \\
\textit{subject to} \quad & 3x_1 + 4x_2 + 2x_3 \leq 7, \\
& 3x_1 - x_2 + 4x_3 = 6, \\
& 6x_1 - 4x_2 + 2x_3 \geq 10, \\
& x_1, x_2, x_3 \geq 0.
\end{aligned}$$

Question:3 **(20 marks)**

(a) Find the initial feasible solution to the following transportation problem by:

(i) ***north-west corner rule,***

(ii) ***Minimum cost rule,***

To

From	1	2	3	4	*Supply*
1	4	3	10	7	15
2	1	0	6	2	25
3	5	8	15	9	10
Demand	15	10	20	5	

(b) **By using *Vogel's approximation method* solve the above problem.**

Question:4 **(20 marks)**

Solve the following *Assignment* problem:

	I	II	III	IV
1	10	5	9	18
2	13	9	6	12
3	3	2	4	4
4	18	9	12	17

WITH THE BEST WISHES

المستوى الرابع - رياضيات - جبر لى / ٢١٥

Mans. Univ. **Faculty of Science** **Dept. Math.**		**Fourth Year Exam.** **Lie algebra Math 415** **Tim 2 Hours**

Answer the following Questions

[1] a) Define what we mean by a map ϕ is a homomorphism from Lie algebra L_1 to a Lie algebra L_2

b) Define the mapping ad : $L \rightarrow g\ell(L)$ by $x \rightarrow ad(x)$ such that $ad(x)(y) = [x,y]$ Prove that ad(x) is a homomorphism called adjoint homomorphism.

c) Prove also that the kernel of ad is the center of L

[2] Let A be an algebra. A derivation D of A is an linear map such that $D(ab) = aD(b) + D(a)b$ for all $a, b \in A$

a) Show that $[D,E] = DoE - EoD$ is a derivation where E is also a derivation.

b) Show also that adx: $L \rightarrow L$ is also derivation

c) Let A be algebra $C^{\infty}R$. Define $D(f) = f'$. Prove that D is a derivation of A.

[3] Define what we mean by L is a solvable Lie algebra.

a) Prove that every sub algebra and homomorphic image of L are solvable

b) Suppose that L has an ideal I such that I and L/I are solvable. Then L is solvable.

[4] Let L be a Lie algebra. Define what we mean by V is L- module, where V is finite dimensional vector space

a) Give an example of a module structure of L by L

b) Define what we mean by a L – module V is irreducible and also reducible.

c) Consider the Lie algebra $Sl(2,\mathbb{C})$, with the basis

$$e = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad f = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad h = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$[e,f] = h$$

$$[e,h] = -2e$$

$$[f,h] = 2f$$

Give a irreducible module structure of $Sl(2,\mathbb{C})$

المستوى الرابع - رياضيات / الاحصاء والحاسب - تحليل عددى (٢) ، ٤١٣ ر

Level: 4 Program: Mathematics + Statistics & Computer Science Numerical Analysis (2) (413ر)	Faculty of Science Mathematics Department	1^{st} Semester Time: 2 hour Date: 12/1/2013

Answer the following Questions:

Question (1)

(i) What are row operations? (3 Marks)

(ii) Determine the LU factorization of the coefficient matrix of the system (8 Marks)

$$\begin{aligned} 3x_1 - 13x_2 + 9x_3 + 3x_4 &= -19 \\ -6x_1 + 4x_2 + x_3 - 18x_4 &= -34 \\ 6x_1 - 2x_2 + 2x_3 + 4x_4 &= 16 \\ 12x_1 - 8x_2 + 6x_3 + 10x_4 &= 26 \end{aligned}$$

(iii) Find the least squares line approximating the data in following Table

x_i	2	4	6	7	12
y_i	1	5	7	10	11

(4 Marks)

Question (2)

(i) Define characteristic polynomial $p(\lambda)$ and the spectral radius $\rho(A)$ of a matrix A. Find $\|Ax\|_p$, $\|A\|_p$, $p = 2, \infty$ (6 Marks)

$$A = \begin{bmatrix} 2 & 1 & 0 \\ -3 & 0 & 1 \\ 1 & -2 & 0 \end{bmatrix}, \qquad x = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

(ii) Determine the trigonometric polynomial that approximates the $f(x) = |x|$ on $[-\pi, \pi]$. (3 Marks)

(iii) Show that the linear system (6 Marks)

$$\begin{aligned} 4x_1 + 3x_2 \quad\quad &= 24 \\ 3x_1 + 4x_2 - x_3 &= 30 \\ - x_2 + 4x_3 &= -24 \end{aligned}$$

has the solution $(3, 4, -5)^t$, and using $x^{(0)} = (1,1,1)^t$ find the first two iterations of the SOR method with the optimal choice of ω ($\rho(T_\omega) = 0.24$). Find the error?

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Question (3)

(i) Define **two** means for measuring the amount by which an approximation to the solution to a linear system differs from the true solution to the system. (3 Marks)

(ii) Solve this system of linear equations: (9 Marks)

$$0.0001\,x + y = 1$$
$$x + y = 2$$

using **(a)** no pivoting, **(b)** partial pivoting, **(c)** and scaled partial pivoting. Carry at most five significant digits of precision (rounding) to see how finite precision computations and round-off errors can affect the calculations.

(iii) Let $T_n(x)$ denote the Chebyshev polynomial, Show that

$$T_{n+1}(x)=2\,x\,T_n(x)-T_{n+1}(x), \quad n>1$$ (3 Marks)

Question (4)

(i) Derive the Legendre polynomials $\{P_n(x)\}$ of degree 2 in $[-1, 1]$ using Gram-Schmidt process. Then find the least-squares polynomial of degree 2 in $[-1, 1]$ for the function $f(x) = e^x$. (5 Marks)

(ii) Prove that the sequence

$$x^{(k)}=T\,x^{(k-1)}+c, \quad k\geq 1$$

is convergent to the unique solution of $x=T\,x+c$ iff $\rho(T)<1$. (5 Marks)

(iii) Show that the following function $G: D\subset\mathbb{R}^2\to\mathbb{R}^2$ has unique fixed point in

$$D=\{(x_1,x_2):0\leq x_1,x_2\leq 1.5\},\ G(x_1,x_2)=\left(\frac{x_1^2+x_2^2+8}{10},\frac{x_1\ x_2^2+x_1+8}{10}\right).$$ (5 Marks)

Best Wishes; *Dr. Tamer Mohamed El-Azab*

المستوى الرابع - رياضيات - ميكانيكا الكم / ٤٢٣

Mansoura Univ.
Faculty of Science
Mathematics Dept.
Subject:Math. 423
Course Quantum Mechanics

4thYear: math.
Date Jan.2013
Time: 2 hours
Full marks: 80

Answer the following questions:

[1] a) Why classical mechanics is not suitable to study small particles (which are affected by emission or absorption of light)? Derive Schrodinger equation.

b) Derive Ehrenfest theorem. Explain its importance. [20 marks].

[2] Solve Schrodinger equation for the harmonic Oscillator. What is the importance of the value 1/2 in the relation E=hw(n+1/2). [20 marks].

[3] a) Write short notice on quantum entanglement. Comment on Einstein objection to quantum mechanics.

b) Explain the WKB approximation method. [20 marks].

[4] a) Derive uncertainty principle for unitary operators.

b) Derive the Casimir operator for the SU(2) algebra of the angular momentum operators [20 marks].

المستوى الرابع - رياضيات + احصاء وحاسب - نظرية الشبكات / ٤١٨

El-Mansoura- Egypt	4^{th} level of Math. Program and	المنصورة – مصر
Mansoura University	the Program of Statistics and Computer Science	جامعة المنصورة
Faculty of Science	Subject: Lattice Theory نظرية الشبكات	كلية العلوم
Mathematics Department	Course Code: Math 418	قسم الرياضيات
First Term. 12 - 13:	**Date: Jan. 2013**	**Time: 2 hours**

Answer the following five questions:

1- a- Give two distinct equivalent definitions of lattices. (6 points)

b- Give an example of each of:

1 - A partially ordered set (poset) but not a meet semilattice. (2 points)

2 - A non- modular lattice with 7 elements. (2 points)

3 - A modular lattice but not distributive with 6 elements. (2 points)

4 - A distributive lattice having n elements for each n. (2 points)

5 - An $\leq$ - homomorphism between two lattices, (2 points)
but not $\vee$- homomorphism.

c- Prove:

1- If $a_1 \leq b_1$ & $a_2 \leq b_2 \Rightarrow a_1 \wedge a_2 \leq b_1 \wedge b_2$. (2 points)

2- If $a \leq c$, then $a \vee (b \wedge c) \leq (a \vee b) \wedge c$ for each lattice. (2 points)

2- a- Let (L, $\vee$) be a join semilattice as an algebra. (8 points)
Find a semilattice (L, $\leq$) as a poset equivalent to (L, $\vee$).

b- Give an example of a $\vee$-semilattice but not $\wedge$-semilattice. (4 points)

c- Give all $\vee$-semilattices with 4-element set. (4 points)

d- Give all lattices with 5 elements. (4 points)

3- a- For a group $\boldsymbol{G} = (G ; \cdot)$, Show that the set of all normal (10 points)
subgroups $N(\boldsymbol{G})$ of $\boldsymbol{G}$ with the inclusion $\subseteq$ forms a modular lattice.

b - Find the set of subgroups $S_{18\mathbb{Z}}(\mathbb{Z})$, of the group of integers (10 points).
$(\mathbb{Z}, +)$ containing $18\mathbb{Z}$. And then give the Hass Diagram of the lattice. $S_{18\mathbb{Z}}(\mathbb{Z})$. Is the lattice $S_{18\mathbb{Z}}(\mathbb{Z})$ distributive? Why?

4- a- Let $\boldsymbol{L} = (L ; \vee , \wedge)$ be a lattice. (8 points)
Prove that:
$\boldsymbol{L}$ is not modular. $\Rightarrow$ $\boldsymbol{L}$ has a sublattice $\cong N_5$.

b - Determine the lattice $I_{\vee}(\boldsymbol{L})$ of all $\vee$-ideals and the lattice Con ($\boldsymbol{L}$)
of all congruences of a lattice $\boldsymbol{L}$ with 3 elements. (12 points)

Examiner : Dr. Magdi H. Armanious **Full Mark: 80 points**