

Mansoura University Faculty of Science Dept. of Mathematics Neural Networks		4 <sup>th</sup> year Time: 2 Hours Date: 1/1/2013 Maximum 60 Marks
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**Answer the following questions:**

**Question #1 [20 Marks]**

- a) State the main advantages of neural networks?
- b) What is the purpose of using activation function? State its different types?
- c) What is the difference between
1. Action potential and activation function
  2. Feedforward and feedback neural networks?
  3. Supervised and unsupervised learning
  4. Digital computers and neural networks
- d) Draw the diagram of:
1. Biological neuron
  2. Artificial neuron
  3. Single layer neural network
  4. Multilayer neural network
  5. Competitive neural network
  6. Hopfield neural Model

**Question #2 [20 Marks]**

- a) Describe how to represent the following characters (L and M) on the computer screen and then design a Hebb net to classify between L and M characters?
- b) Construct an architecture graph that describes the following:
1. A fully connected feedforward network has 8 source nodes, 2 hidden layers, one with 3 neurons and the other with 2 neurons, and a single out put neuron.
  2. A feedback neural network has 3 source nodes, 2 hidden neurons, and 4 output neurons.
- c) An odd sigmoid function is given by:

$$\Phi(x) = \frac{x}{\sqrt{1+x^2}}$$

State the limits of this function? show that the derivative of  $\Phi(x)$  with respect to  $x$  is given by:

$$\frac{\partial \Phi}{\partial x} = \frac{\Phi^3(x)}{x^3}$$

What is the value of this derivative at the origin?

بغيره على الخلف ←

d) Consider the pseudolinear activation function  $\Phi(x)$  shown in Fig. 1.

1. Formulate  $\Phi(x)$  as a function of  $x$ .
2. What happens to  $\Phi(x)$  if  $a$  is allowed to approach zero?

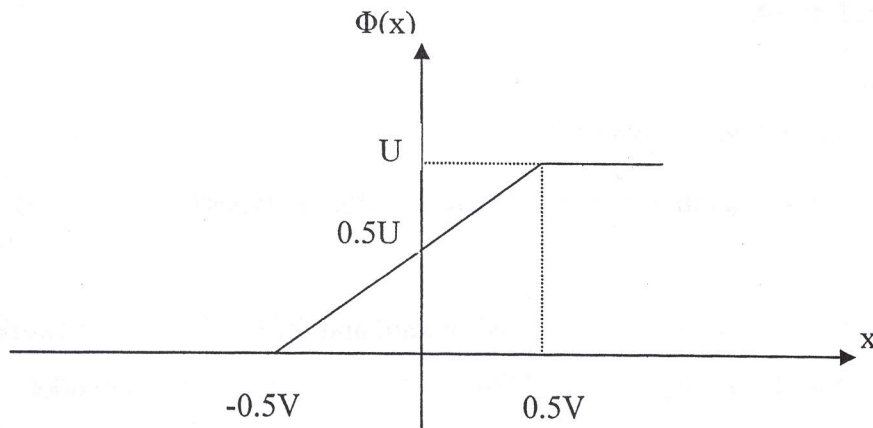


Fig. 1

**Question #3 [20 Marks]**

a) Design a neural network to perform the following logic operations NAND, OR, XOR?

b) Realize the following equation by using neural networks:

$$Z = \frac{3}{\sqrt{18X + 9Y}} + \frac{4}{(8X - 4Y)^2}$$

Draw the architecture of the network and state the values of weights and activation functions?

c) A 2-2-2-1 feedforward neural network has the following weight specification:

between the input and first hidden layer:

$$w_{11}=4 \quad w_{12}=1.5 \quad w_{21}=2 \quad w_{22}=-2$$

between the first and second hidden layer:

$$w_{11}=2 \quad w_{12}=3 \quad w_{21}=-3 \quad w_{22}=4$$

between the second hidden layer and output layer:

$$w_{11}=-1 \quad w_{21}=1$$

1. Draw the network structure (write the values of weight between connections).
2. Suppose that all neurons operate in their linear region. Write the input output mapping defining the network.
3. Assume that the activation function is sigmoid for all neurons. Write the input output mapping defining this new network.

امتحان الفصل الدراسي الاول ٢٠١٣ م  
برنامج : إحصاء وعلوم الحاسب  
المستوى: الرابع  
اسم المقرر : نظرية إحصائية ( ٢ )  
كود المادة : ر ٤٣١



٤٣١ /

جامعة المنصورة - كلية العلوم  
قسم الرياضيات  
التاريخ : ١٥ / ١ / ٢٠١٣ م  
الدرجة الكلية : ٨٠ درجة  
الزمن : ساعتان

أجب عن الأسئلة الآتية :-

السؤال الأول: ( أ ) أخذت عينة عشوائية مكونة من 200 وصفة طبية بإحدى المستشفيات وجد من بينهم 80 وصفة تحتوى على عقار البنادول. اختبر الفرض القائل بأن نسبة الوصفات التى بها عقار البنادول هي % 50 (١٠ درجات) (ب) سحبت عينة عشوائية حجمها n ووسطها الحسابى  $\bar{X}$  من مجتمع له توزيع طبيعى وسطه  $\mu$  و تباينه يساوى الواحد الصحيح :

(١) أوجد k بحيث تمثل  $\bar{X} > k$  منطقة رفض ذات حجم 0.05 لاختبار  $H_0: \mu = \mu_0$  ضد  $H_1: \mu > \mu_0$

(٢) أوجد أقل حجم عينة ممكن حتى تكون  $\beta \leq 0.05$  مع بقاء  $\alpha = 0.05$  عند اختبار  $H_0: \mu = 10$

(١٥ درجة)

ضد  $H_1: \mu = 11$

السؤال الثانى: ( أ ) اذكر اوجه الشبه والاختلاف بين اختبار مربع كاي للاستقلال واختبار مربع كاي للتجانس ثم اشرح بالتفصيل اختبار مربع كاي للاستقلال . (١٥ درجة)  
(ب) اذكر اوجه الشبه والاختلاف بين اختبار فرض احصائى حول الوسط الحسابى لمجتمع ما و بين % 100 (1 -  $\alpha$ ) فترة ثقة للوسط الحسابى لنفس المجتمع . (١٥ درجة)

السؤال الثالث:

( أ ) اشرح بالتفصيل خطوات اختبار فرض احصائى حول الفرق بين نسبتي مجتمعين (١٠ درجات)  
(ب) الجدول التكرارى التالى يوضح اوزان عينة عشوائية مكونة من 80 طالب فى المرحلة الابتدائية :

فئات الوزن	8-	10-	12-	14-	16-18
التكرار	12	18	20	18	12

هل يمكن القول بان اوزان الطلاب تتبع التوزيع الطبيعى ؟ استخدم  $\alpha = 0.05$  (١٥ درجة)

$$Z_{0.05} = 1.645 , Z_{0.025} = 1.96 , t_{(0.025, 10)} = 2.23 , t_{(0.025, 9)} = 2.26 ,$$


$$\chi^2_{(0.025, 3)} = 9.35 , \chi^2_{(0.975, 3)} = 0.22 , \chi^2_{(0.025, 2)} = 7.38 , \chi^2_{(0.975, 2)} = 0.05$$

مع أطيب التمنيات بالتوفيق

( اقلب الورقة )

د. محمد جاد



<p>Level: 4 Program: Mathematics + Statistics &amp; Computer Science Numerical Analysis (2) (413)</p>	 Faculty of Science Mathematics Department	<p>1<sup>st</sup> Semester Time: 2 hour Date: 12/1/2013</p>
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Answer the following Questions.

Question (1)

(i) What are row operations? (3 Marks)

(ii) Determine the LU factorization of the coefficient matrix of the system (8 Marks)

$$\begin{aligned} 3x_1 - 13x_2 + 9x_3 + 3x_4 &= -19 \\ -6x_1 + 4x_2 + x_3 - 18x_4 &= -34 \\ 6x_1 - 2x_2 + 2x_3 + 4x_4 &= 16 \\ 12x_1 - 8x_2 + 6x_3 + 10x_4 &= 26 \end{aligned}$$

(iii) Find the least squares line approximating the data in following Table

$x_i$	2	4	6	7	12
$y_i$	1	5	7	10	11

(4 Marks)

Question (2)

(i) Define characteristic polynomial  $p(\lambda)$  and the spectral radius  $\rho(A)$  of a matrix A. Find  $\|Ax\|_p$ ,  $\|A\|_p$ ,  $p=2, \infty$  (6 Marks)

$$A = \begin{bmatrix} 2 & 1 & 0 \\ -3 & 0 & 1 \\ 1 & -2 & 0 \end{bmatrix}, \quad x = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

(ii) Determine the trigonometric polynomial that approximates the  $f(x) = |x|$  on  $[-\pi, \pi]$ . (3 Marks)

(iii) Show that the linear system (6 Marks)

$$\begin{aligned} 4x_1 + 3x_2 &= 24 \\ 3x_1 + 4x_2 - x_3 &= 30 \\ -x_2 + 4x_3 &= -24 \end{aligned}$$

has the solution  $(3, 4, -5)^t$ , and using  $x^{(0)} = (1, 1, 1)^t$  find the first two iterations of the SOR method with the optimal choice of  $\omega$  ( $\rho(T_\omega) = 0.24$ ). Find the error?

### Question (3)

- (i) Define two means for measuring the amount by which an approximation to the solution to a linear system differs from the true solution to the system. (3 Marks)

- (ii) Solve this system of linear equations. (9 Marks)

$$0.0001x + y = 1$$

$$x + y = 2$$

using (a) no pivoting, (b) partial pivoting, (c) and scaled partial pivoting. Carry at most five significant digits of precision (rounding) to see how finite precision computations and round-off errors can affect the calculations.

- (iii) Let  $T_n(x)$  denote the Chebyshev polynomial, Show that

$$T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x), \quad n > 1 \quad (3 \text{ Marks})$$

### Question (4)

- (i) Derive the Legendre polynomials  $\{P_n(x)\}$  of degree 2 in  $[-1, 1]$  using Gram-Schmidt process. Then find the least-squares polynomial of degree 2 in  $[-1, 1]$  for the function  $f(x) = e^x$ . (5 Marks)

- (ii) Prove that the sequence

$$x^{(k)} = T x^{(k-1)} + c, \quad k \geq 1$$

is convergent to the unique solution of  $x = T x + c$  iff  $\rho(T) < 1$ . (5 Marks)

- (iii) Show that the following function  $G: D \subset \mathbb{R}^2 \rightarrow \mathbb{R}^2$  has unique fixed point in

$$D = \{(x_1, x_2) : 0 \leq x_1, x_2 \leq 1.5\}, \quad G(x_1, x_2) = \left( \frac{x_1^2 + x_2^2 + 8}{10}, \frac{x_1 x_2^2 + x_1 + 8}{10} \right). \quad (5 \text{ Marks})$$

*Best Wishes;*

*Dr. Tamer Mohamed El-Azab*



Mansoura university  
Faculty of science  
Math. Depart.

Final Exam 1-st term  
2012-2013  
Program: Comp. Science & Statistics

Subject. Stochastic processes  
Time: 2 hours  
Date: 25/12/2012

Answer the following questions

**Q1: ( 20 marks )**

- (a) Three coins are placed in a row on a table. At each stage, a coin selected at random and turned over. Let  $X_n$  denotes the total number of heads out of  $n$ th trial.
- 1- Show that  $\{X_n, n \geq 1\}$  is a Markov chain.
  - 2- Find its transition probabilities matrix.
- (b) Give an example of stochastic process with continuous state space and discrete parameter space.

**Q2: ( 20 marks )**

- (a) Define renewal process  
(b) Let  $\{X_n, n = 0, 1, 2, \dots\}$  is a Markov chain. Prove the Chapman-Kolmogorov equation

$$P_{ij}^{(n+m)} = \sum_{k=0}^{\infty} P_{ik}^n P_{kj}^m.$$

- (c) Suppose that if it rains today, then it will rain tomorrow with probability 0.4, if it dry today, then it will rain tomorrow with probability 0.8.
- (i) Find the probability that 3 October is a dry day ,given that 1 October is a rain day
  - (ii) If we know that 1 October is a dry day with probability 0.25, find the probability that it will rain on 3 October.

**Q3: ( 20 marks )**

- (a) If  $N(t)$  is a Poisson process and  $s < t$ , then find  $P(N(s) = k / N(t) = n)$ .
- (b) Suppose that people immigrate into a territory at a Poisson rate  $\lambda = 1$  per day
- (i) What is the probability that time until the second immigrant arrives 2 days at most?
  - (ii) What is the probability that elapsed time between tenth and eleventh arrival exceeds 3 days?

**Q4: ( 20 marks )**

- (a) State two equivalent definitions for nonhomogenous Poisson process.
- (b) Suppose that families migrate to an area at a Poisson rate 3 families per week. If the number of people in each family is independent and takes on the values 1, 2, 3 with respective probabilities  $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ . Find the expected value and variance of the number of individuals migrating to this area during a fixed six week period

Good luck

El-Mansoura- Egypt	4 <sup>th</sup> level of Math. Program and	المنصورة - مصر
Mansoura University	the Program of Statistics and Computer Science	جامعة المنصورة
Faculty of Science	Subject: Lattice Theory	كلية العلوم
Mathematics Department	Course Code: Math 418	قسم الرياضيات
First Term. 12 - 13:	Date: Jan. 2013	Time: 2 hours

**Answer the following five questions:**

- 1- a- Give two distinct equivalent definitions of lattices. (6 points)
- b- Give an example of each of:
- 1 - A partially ordered set (poset) but not a meet semilattice. (2 points)
  - 2 - A non- modular lattice with 7 elements. (2 points)
  - 3 - A modular lattice but not distributive with 6 elements. (2 points)
  - 4 - A distributive lattice having  $n$  elements for each  $n$ . (2 points)
  - 5 - An  $\leq$  - homomorphism between two lattices, (2 points)  
but not  $\vee$ - homomorphism.
- c- Prove:
- 1- If  $a_1 \leq b_1$  &  $a_2 \leq b_2 \Rightarrow a_1 \wedge a_2 \leq b_1 \wedge b_2$ . (2 points)
  - 2- If  $a \leq c$ , then  $a \vee (b \wedge c) \leq (a \vee b) \wedge c$  for each lattice. (2 points)
- 2- a- Let  $(L, \vee)$  be a join semilattice as an algebra. (8 points)  
Find a semilattice  $(L, \leq)$  as a poset equivalent to  $(L, \vee)$ .
- b- Give an example of a  $\vee$ -semilattice but not  $\wedge$ -semilattice. (4 points)
- c- Give all  $\vee$ -semilattices with 4-element set. (4 points)
- d- Give all lattices with 5 elements. (4 points)
- 3- a- For a group  $G = (G ; \cdot)$ , Show that the set of all normal (10 points)  
subgroups  $N(G)$  of  $G$  with the inclusion  $\subseteq$  forms a modular lattice.
- b - Find the set of subgroups  $S_{18\mathbb{Z}}(\mathbb{Z})$ , of the group of integers (10 points).  
 $(\mathbb{Z}, +)$  containing  $18\mathbb{Z}$ . And then give the Hass Diagram of the  
lattice.  $S_{18\mathbb{Z}}(\mathbb{Z})$ . Is the lattice  $S_{18\mathbb{Z}}(\mathbb{Z})$  distributive? Why?
- 4- a- Let  $L = (L ; \vee, \wedge)$  be a lattice. (8 points)  
Prove that:  
 $L$  is not modular.  $\Rightarrow L$  has a sublattice  $\cong N_5$ .
- b - Determine the lattice  $I_{\vee}(L)$  of all  $\vee$ -ideals and the lattice  $\text{Con}(L)$   
of all congruences of a lattice  $L$  with 3 elements. (12 points)



Mansoura university	1 <sup>st</sup> term	المقرر: عمليات عشوائية (١)
Faculty of science	2011/2012	الزمن: ساعتان
Math. Depart	4 <sup>th</sup> year	التاريخ: ٢٠١١/١٢/٣١

**Answer the following questions**

**Q1: ( 20 marks)**

(a) Define the following

Stochastic process-Markov chain- Stationary Markov chain (9 marks)

(b) Let  $\{N_1(t), t \geq 0\}$  and  $\{N_2(t), t \geq 0\}$  be two Poisson processes having

respectively rates  $\lambda_1, \lambda_2$  and let  $N(t) = N_1(t) + N_2(t)$ , then show that

$\{N(t), t \geq 0\}$  is a Poisson process with mean  $(\lambda_1 + \lambda_2)t$ . (6 marks)

(c) If  $\{N(t), t \geq 0\}$  is a Poisson process and  $s < t$ , then for any  $k$  find

$$P \left[ N(s) = k / N(t) = n \right] \quad (5 \text{ marks})$$

**Q2: ( 20 marks)**

Let  $\{X_n, n = 0, 1, 2, \dots\}$  is a Markov chain with state space  $S = \{0, 1, 2\}$  has a

transition probability matrix  $P = \begin{bmatrix} 0.1 & 0.2 & 0.7 \\ 0.8 & 0.1 & 0.1 \\ 0.1 & 0.8 & 0.1 \end{bmatrix}$  and starting vector

$\alpha_0 = (0.2, 0.5, 0.3)$ . Find

(i)  $P(X_3 = 2)$  (ii)  $P(X_0 = 0, X_1 = 1, X_2 = 2)$  (iii)  $P(X_5 = 2 | X_2 = 1)$

**Q3: ( 20 marks)**

Consider a Markov chain with state space  $S = \{0, 1, 2\}$  and transition

probability matrix  $P = \begin{bmatrix} \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{4} & \frac{2}{3} & \frac{1}{4} \\ \frac{1}{6} & \frac{1}{3} & \frac{1}{2} \end{bmatrix}$ . Find the stationary distribution for it if

exists.

**Q4: ( 20 marks)**

(a) Consider a Poisson process  $\{N(t), t \geq 0\}$  with rate  $\lambda$ , and let  $T_n$  denote the elapsed time between the  $(n-1)$ -th and the  $n$ -th event. Find the probability distribution of  $T_n$ . (10 marks)

(b) Suppose that people immigrate into a territory at a Poisson rate  $\lambda = 2$  per day

(i) What is the expected time until the eleventh immigrant arrives?

(ii) What is the probability that elapsed time between ninth and tenth arrival exceeds 3 days? (10 marks)

امتحان دور يناير ٢٠١٢  
برنامج : إحصاء وعلوم الحاسب  
المستوى: الرابع  
اسم المقرر : نظرية إحصائية ( ٢ )  
كود المادة : ٤٣١



جامعة المنصورة - كلية العلوم  
قسم الرياضيات  
التاريخ : ٢١ - ١ - ٢٠١٢ م  
الدرجة الكلية : ٨٠ درجة  
الزمن : ساعتان  
المستوى الرابع  
إحصاء رياضي  
نظرية إحصائية  
( ٤٣١ )

أجب عن الأسئلة الآتية :-

السؤال الأول: (أ) اشرح اختبار نسبة الاحتمال المتوالى (١٠ درجات)  
(ب) أخذت عينة عشوائية ذات حجم  $n$  من مجتمع طبيعي وسطه الحسابي يساوى صفر و تباينه  $\theta$  حيث  $\theta$  مقدار ثابت موجب مجهول . أوجد أفضل منطقة رفض ذات حجم  $\alpha$  لاختبار فرض العدم  $H_0: \theta = \theta_0$  مقابل الفرض البديل  $H_1: \theta > \theta_0$  حيث  $\theta_0$  ثابت موجب . (١٥ درجة)

السؤال الثاني: (أ) اشرح بالتفصيل خطوات اختبار فرض إحصائي حول الفرق بين وسطي مجتمعين سواء كان تباين المجتمعين معلوم أو غير معلوم. (١٠ درجات)  
(ب) إذا كان  $X$  متغير عشوائي يتبع التوزيع الأسى ببارامتر  $\theta$  . قارن بين المنطقتين الحرجتين  $C_1 = \{x: x > 1\}$  ,  $C_2 = \{x: x < 0.0725\}$  للاختبارين المتنافسين  $T_1$  ,  $T_2$  لاختبار فرض العدم  $H_0: \theta = 2$  ضد الفرض البديل  $H_1: \theta < 2$  (يمكن الاكتفاء بمشاهدة واحدة) (١٥ درجة)

السؤال الثالث: (أ) اذكر الفرق بين الاختبارات البارامترية (المعلمية) و الاختبارات الغير بارامترية (اللامعلمية) ثم تكلم عن اختبار مربع كاي للاستقلال موضحا سبب أنه ذات طرف أيمن فقط . (١٥ درجة)  
(ب) أخذت عينة عشوائية مكونة من عشرة أشخاص و طبق عليهم برنامج معين لإنقاص الوزن لمدة شهر و سجلت أوزانهم بالكجم قبل و بعد البرنامج فكانت النتائج كالتالي :

الوزن قبل البرنامج	62	82	77	57	62	90	82	42	95	60
الوزن بعد البرنامج	53	73	65	55	67	85	79	42	80	60

بفرض أن الأوزان قبل و بعد تطبيق البرنامج تتبع التوزيع الطبيعي . هل يمكن القول أن هذا النظام قد أفاد في إنقاص الوزن ؟ استخدم مستوى معنوية  $\alpha = 0.05$  (١٥ درجة)

$$Z_{0.05} = 1.645 , Z_{0.025} = 1.96 , t_{(0.025,10)} = 2.23 , t_{(0.025,9)} = 2.26 , \\ t_{(0.05,9)} = 1.83 , t_{(0.05,10)} = 1.81$$

مع أطيب التمنيات بالتوفيق  
د. محمد جاد

El-Mansoura- Egypt	4 <sup>th</sup> Level .	المنصورة - مصر
Mansoura University	Program: Statistics and Computer Science	جامعة المنصورة
Faculty of Science	Subject: Lattice Theory	كلية العلوم
Mathematics Department	Course Code: Math. 418	قسم الرياضيات
First Term: Jan 2012	Date: 14 Jan. 2012	Time: 2 hours

**Answer the following five questions:**

- 1- Give two distinct equivalent definitions of lattices. (5 points)  
 And then give an example of each of: (each item 3 points)
- a- A partially ordered set (poset) but not a lattice,
  - b- The smallest non- modular lattice,
  - c- A modular lattice but not distributive,
  - d- A distributive lattice having more than 4 elements,
  - e- An  $\leq$  - homomorphism between two lattices but not  $\vee$ - homomorphism.
- 2- a- Let  $(L, \wedge)$  be a meet semilattice as an algebra. (10 points)  
 Find a semilattice  $(L, \leq)$  as a poset.  
 Give an example of a  $\wedge$ -semilattice but not  $\vee$ -semilattice.
- b- Give all  $\vee$ -semilattices with 4-element set. (5 points)
- c- Give an example of a modular lattice, but not distributive with 7 elements. (5 points)
- 3- For a commutative group  $G = (G ; \cdot)$ .
- a- Show that the set of all subgroups  $S_N(G)$  (6 points)  
 containing a subgroup  $N$  of  $G$  forms a lattice.
- b- Find the set of subgroups  $S_{12Z}(Z)$  of the group (6 points).  
 of integers  $(Z, +)$ .
- c- Give the Hass Diagram of the lattice.  $S_{12Z}(Z)$ . (8 points)  
 Is the lattice  $S_{12Z}(Z)$  distributive? *Why?*
- 4- a- Let  $a, x, y$  be any three elements in a lattice  $L = (L ; \vee, \wedge)$ . (10 points)  
 Prove that:  
 $L$  is modular  $\Leftrightarrow$  “  $a \wedge x = a \wedge y$  &  $a \vee x = a \vee y$  &  $x \leq y \Rightarrow x = y$  “.
- b- Give two equivalent definitions of a  $\vee$ -ideal of a lattice (4 points)  
 $L = (L ; \vee, \wedge)$ .
- c - Define a congruence relation  $\theta$  on a lattice  $L = (L ; \vee, \wedge)$ . (6 points)  
 And show that each congruence class  $[a]\theta$  is a convex sublattice.

4<sup>th</sup> Level Examination  
Math 413  
Numerical Analysis 2  
Time : 2 hours  
17/1/2012

٤١٣ ر  
تحليل عددي ٢  
شعبة: الرياضيات + الإحصاء  
وعلوم الحاسب

Mansoura University  
Faculty of science  
Department of mathematics

**Answer the following questions (20 marks each)**

Q<sub>1</sub>.

(i) Solve the system

$$\begin{aligned}2x_1 - x_2 + x_3 &= -1, \\3x_1 + 3x_2 + 9x_3 &= 0, \\3x_1 + 3x_2 + 5x_3 &= 4,\end{aligned}$$

using the LU-decomposition.

(ii) Find the linear least-squares polynomial to approximate the following data

$x_i$	2	4	6	8
$y_i$	2	11	28	40

(iii) Show that the boundary value problem

$$\begin{aligned}y'' &= -\frac{2}{x}y' + \frac{2}{x^2}y + \frac{\sin(\ln(x))}{x^2}, \quad 1 \leq x \leq 2, \\y(1) &= 1, \quad y(2) = 2,\end{aligned}$$

has a unique solution in  $D = \{(x, y, y') \mid 1 \leq x \leq 2, -\infty < y < \infty, -\infty < y' < \infty\}$ ,

Then explain how to use the linear shooting method to approximate  $y(1.1)$  [take  $h = 0.1$ ]

Q<sub>2</sub>.

(i) Define the spectral radius  $\rho(A)$  of matrix  $A$ .

(ii) Apply Gauss-Siedel method to find the second approximation of the solution of the system

$$\begin{aligned}10x_1 - x_2 + 2x_3 &= 6, \\-x_1 + 11x_2 - x_3 + 3x_4 &= 25, \\2x_1 - x_2 + 10x_3 - x_4 &= -11, \\3x_2 - x_3 + 8x_4 &= 15,\end{aligned}$$

take  $x^{(0)} = (0,0,0,0)^t$ .

4<sup>th</sup> Level Examination  
Math 413  
Numerical Analysis 2  
Time : 2 hours  
17/1/2012

٤١٣ ر  
تحليل عددي ٢  
شعبة: الرياضيات + الإحصاء  
وعلوم الحاسب

Mansoura University  
Faculty of science  
Department of mathematics

**Answer the following questions (20 marks each)**

Q<sub>1</sub>.

(i) Solve the system

$$\begin{aligned}2x_1 - x_2 + x_3 &= -1, \\3x_1 + 3x_2 + 9x_3 &= 0, \\3x_1 + 3x_2 + 5x_3 &= 4,\end{aligned}$$

using the LU-decomposition.

(ii) Find the linear least-squares polynomial to approximate the following data

$x_i$	2	4	6	8
$y_i$	2	11	28	40

(iii) Show that the boundary value problem

$$\begin{aligned}y'' &= -\frac{2}{x}y' + \frac{2}{x^2}y + \frac{\sin(\ln(x))}{x^2}, \quad 1 \leq x \leq 2, \\y(1) &= 1, \quad y(2) = 2,\end{aligned}$$

has a unique solution in  $D = \{(x, y, y') \mid 1 \leq x \leq 2, -\infty < y < \infty, -\infty < y' < \infty\}$ ,

Then explain how to use the linear shooting method to approximate  $y(1.1)$  [take  $h = 0.1$ ]

Q<sub>2</sub>.

(i) Define the spectral radius  $\rho(A)$  of matrix  $A$ .

(ii) Apply Gauss-Siedel method to find the second approximation of the solution of the system

$$\begin{aligned}10x_1 - x_2 + 2x_3 &= 6, \\-x_1 + 11x_2 - x_3 + 3x_4 &= 25, \\2x_1 - x_2 + 10x_3 - x_4 &= -11, \\3x_2 - x_3 + 8x_4 &= 15,\end{aligned}$$

take  $x^{(0)} = (0,0,0,0)^t$ .

(iii) Prove that the sequence

$$x^k = Tx^{k-1} + c, \quad k \geq 1$$

is convergent to the unique solution of  $x = Tx + c$  iff  $\rho(T) < 1$

(iv) Derive the SOR procedure to accelerate the convergence of Gauss-Siedel method.

---

Q3.

(i) Derive the Legendre polynomials  $\{P_n\}$  of degree 2 in  $[-1,1]$  using Gram-Schmidt process. Then find the least-squares polynomial of degree 2 in  $[-1,1]$  for the function  $f(x) = e^{-x}$ .

(ii) Let  $T_n(x)$  denote the Chebyshev polynomial:

- show that  $T_{n+1}(x) = 2xT_n(x) - T_{n-1}(x)$ ,  $n \geq 1$ .

- Show that  $\{T_n\}$  are orthogonal in  $[-1,1]$  with respect to  $\omega(x) = \frac{1}{\sqrt{1-x^2}}$ .

(iii) Find the trigonometric least-square polynomial of degree two for  $f(x) = x$  on  $[-\pi, \pi]$ .

---

Best wishes

Prof. E. M. Elabbasy

2012-13 - 15/1/2012 - 3 hours - 67.5 marks

**Faculty of Sciences**  
**Department of Mathematics**  
**Course: Operations researches**  
**Date: 15/1/2012**  
**Examiner: Dr Sameh Askar**



**Year: 4<sup>th</sup> (Statistics and Computer sciences)**  
**Full mark: 67.5**  
**Time: 3 hours**  
**Semester: January**

**Answer the following questions**

**Question 1-a:** Let  $f: \Omega \rightarrow \mathbb{R}, f \in C^2$  be defined on an open set  $\Omega \subseteq \mathbb{R}^n$  then  $f$  is convex on  $\Omega \Leftrightarrow \forall x \in \Omega$  the Hessian  $H(x)$  of  $f$  at  $x$  is positive semi-definite matrix. (7 marks)

[b] Based on the fact says that: if  $g(x)$  is strictly convex and  $f(x)$  is strictly convex, where  $x \in \mathbb{R}^n$  then  $(f \circ g)(x)$  is strictly convex. Show that  $f(x_1, x_2, x_3) = e^{x_1^2 + x_2^2 + x_3^2}$  is strictly convex. (5 marks)

[c] Define the following: epigraph, convex function. Explain the geometric meaning of a convex function. (4 marks)

**Question 2-a:** Define the following: Local minima, global minima, saddle point. (3 marks)

[b] Prove that if  $L(x, \mu)$  has a saddle point  $(x_0, \mu_0)$  for all  $\mu \geq 0$  then  $g(x_0) \leq 0, \mu_0 g(x_0) = 0$  for the NLPP of Minimizing  $f(x)$  subject to  $g_i(x) \leq 0, i = 1, 2, \dots, m$ . (5 marks)

[c] Use Kuhn-Tucker conditions to solve the following NLPP:

$$\begin{aligned} \text{Minimize} \quad & f(x) = x_1^2 + x_2^2 + x_3^2 + 40x_1 + 20x_2 - 3000 \\ \text{s. t.} \quad & x_1 \geq 50, \\ & x_1 + x_2 \geq 100, \quad x_1 + x_2 + x_3 \geq 150 \end{aligned} \quad (8 \text{ marks})$$

**Question 3-a:** Write down Kuhn-Tucker conditions for the QPP defined as follows:

$$\begin{aligned} \text{Max } f(x_1, x_2, \dots, x_n) &= \sum_{j=1}^n c_j x_j + \frac{1}{2} \sum_{j=1}^n \sum_{k=1}^n c_{jk} x_j x_k \\ &\sum_{j=1}^n a_{ij} x_j \leq b_i, \quad i = 1, 2, \dots, m \\ &x_j \geq 0, \quad j = 1, 2, \dots, n \\ &c_{jk} = c_{kj}, \quad \forall j, k \end{aligned} \quad (5 \text{ marks})$$

[b] Use Wolfe's algorithm to solve:  $\text{Max } z = 4x_1 + 2x_2 - x_1^2 - x_2^2 - 5$  subject to:

$$x_1 + x_2 \leq 4, x_1, x_2 \geq 0. \quad (7.5 \text{ marks})$$

[c] Under what conditions should the following general quadratic form be convex?

$$f(x) = cx + \frac{1}{2} x^T Q x$$

where,  $x^T, c \in \mathbb{R}^n$  and  $Q$  is a symmetric  $n \times n$  real matrix. (5 marks)

**Question 4-a:** Use Beale's algorithm to solve:  $\text{Max } z = 2x_1 + 3x_2 - x_1^2$  subject to:

$$x_1 + 2x_2 \leq 4, x_1, x_2 \geq 0. \quad (7 \text{ marks})$$

[b] Suppose you have a bi-objective function with a set of solutions,  $A = (8,5), B = (9,2), C = (12,1), D = (11,2)$  and  $E = (16,2)$ . Define the domination principle and apply it on those solutions to find out dominated and non-dominated solutions with their ranks. (5 marks)

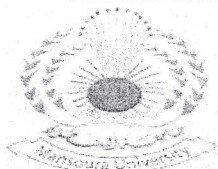
[c] Show that the solutions of the following problem (Binh problem):

$$\begin{aligned} \text{Minimize} \quad & \begin{cases} f_1(x_1, x_2) = x_1^2 + x_2^2 \\ f_2(x_1, x_2) = (x_1 + 5)^2 + (x_2 + 5)^2 \end{cases} \\ \text{Subject to} \quad & -5 \leq x_1 \leq 10, -5 \leq x_2 \leq 10 \end{aligned}$$

obtained by the weighted-sum approach depend on the weight  $w_2$ . Plot five solutions on the trade-off surface for  $w_2 \in [0, 1]$  step 0.2. (6 marks)

*Best wishes and good luck, the examiner...*

Faculty of Sciences  
 Department of Mathematics  
 Course: Linear programming  
 Date: 24/12/2011  
 Course code: (M421)



Year: level 4 (Math./Stat.)  
 Full mark: 80  
 Time: 2 hours  
 Semester: January

Answer the following questions

Question I-a: Mark true or false and correct the false ones

1. Any hyperplane divides  $\mathbb{R}^n$  into two open half spaces.
2. A hyperplane is a closed and convex set.
3. Any continuous function on a compact set attains its minimum on the set.
4. The intersection of an infinite number of closed half spaces is called a polytope.
5. If  $A = \{x_1, x_2\}$  then  $H_{co}(A) = \{x: x = \alpha x_1 + (1 - \alpha)x_2, 0 < \alpha < 1\}$ .
6. A simplex in n-dimension has  $n + 1$  vertices.
7. A matrix is called positive-semi definite if  $z^t A z > 0$  for all  $z \neq 0$ .
8. The linear function  $z = 2x_1 + 4x_2$  attains its extreme values at the vertices.
9. A feasible solution is a solution on which at least one of the constraints is violated.
10. The set of all feasible solutions forms a strictly convex set.
11. The simplex method solves any sort of linear programming problems.
12. A basic solution is a solution for the system of equations  $Ax_B = b$ .
13. Any basic solution is basic feasible solution.
14. Any feasible solution can be reduced to a basic feasible solution.
15. A convex combination of a finite number of different optimum solutions is also an optimum solution.
16. The simplex approach was developed in the late summer of 1947.
17. For only L.P.P.,  $\text{Min } z = -\text{Max } \{z\}$ .
18. The dual of dual is primal.

[15 marks]

b:- The strategic bomber command receives instruction to interrupt the enemy tank production. The enemy has four key plants located in separate cities, and destruction of any one plant will effectively halt the productions of tanks. There is an acute shortage of fuel, which limits the supply to 45000 litres for this particular mission. Any bomber sent to any particular city must have at least enough fuel for the round trip plus 100 litres. The number of bombers available to the commander and their descriptions are as follows:

Bomber type	Description	Km/litre	Number available
A	Heavy	2	40
B	Medium	2.5	30

Information about the location of the plants and their probability of being attacked by a medium bomber and a heavy bomber is given below.

plant	Distance from base (km)	Probability of destruction by	
		A heavy bomber	medium bomber
1	400	0.10	0.08
2	450	0.20	0.16
3	500	0.15	0.12
4	600	0.25	0.20

Formulate it as a L.P.P. to see how many of each type of bombers should be dispatched, and how should they be allocated among the four targets in order to maximize the probability of success?

Please see overleaf

[5 marks]



**Question 2-a:** Prove that a function  $f: \Omega \rightarrow \mathbb{R}$  defined on a convex set  $\Omega \in \mathbb{R}^n$  is convex if and only if for all  $x, y \in \Omega, \lambda \in [0,1]$  then  $f(\alpha x + (1 - \alpha)y) \leq \alpha f(x) + (1 - \alpha)f(y)$ . Interpret the geometric meaning of convex function. [7 marks]

**b:-** Solve graphically the following L.P.P.: **Max**  $z = x_1 + x_2$  subject to  $-2x_1 + x_2 \leq 1, x_1 \leq 2$  and  $x_1 + x_2 \leq 3, x_1, x_2 \geq 0$ . [5 marks]

**c:-** State and prove the reduction of feasibility theorem. [8 marks]

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**Question 3-a:** Use the big M-method to solve: **Max**  $z = 3x_1 + 2x_2$  subject to  $2x_1 + x_2 \leq 2,$   
 $3x_1 + 4x_2 \geq 12, x_1, x_2 \geq 0$ . [10 marks]

**b:-** Use the simplex method to solve:

$$\begin{array}{ll} \text{Max} & z = 2x_1 + 3x_2 \\ \text{Subject to} & -x_1 + 2x_2 \leq 4 \\ & x_1 + 3x_2 \leq 9 \\ & x_1 + x_2 \leq 6 \\ & x_1, x_2 \text{ Unrestricted} \end{array} \quad \text{[10 marks]}$$

**Question 4:** Given the objective function:  $f(x) = 6x_1 + 7x_2 + 3x_3 + 5x_4$  and the inequality constraints,

$$\begin{aligned} 5x_1 + 6x_2 - 3x_3 + 4x_4 &\geq 12, \\ x_2 + 5x_3 - 6x_4 &\geq 10, \\ 2x_1 + 5x_2 + x_3 + x_4 &\geq 8, \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

Find the optimum basis feasible solution that gives a minimum value for the objective function by using the dual simplex method. [20 marks]

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**Best wishes and good luck...**

**Dr Sameh Askar**

Mansoura University Faculty of Science Physics Department	4 <sup>th</sup> Year-Physics	First Semester final Exam, 2011-2012 January, 2012 (22/01/2012) Time: 3 Hours
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Physics (4b)

**FORTRAN Language and Quantum Mechanics II**

Answer (FIVE) Only of the following Questions:

Total Mark: 90 Marks

**FORTRAN Language**

1)	<p>a) What will be the values of X, Y and L after the execution of the following statements:</p> <pre> Y = 3.0 X = 2.0 L = 1 10 GOTO (20,30,40,50,50,50), L 20 L=L+5    X=X+2.0    Y=Y+X    GOTO 10 30 X=X- 6.0    Y=Y- 49.    L=L+10    GOTO 60 40 X=X + 2.0    Y=Y + X    L=L-1    GOTO 10 50 X=X + 4.0    Y=Y + X    L=L-1    GOTO 10 60 CONTINUE    END </pre>	<p>b) Determine the values of Y, X, and J after execution of the following:</p> <pre> Y=2.0 X=3.0 DO 10 J=1,9,2 IF(J*2.LE.15)X=X+3.0 Y=Y+X 10 CONTINUE END </pre>	18 each (9)
2.a)	<p>Write the following expressions in FORTRAN FORM:</p> <p>i- <math>B = \frac{e^{x/\sqrt{2}} \cos(\sqrt{x/2} + \pi/8)}{\sqrt{2\pi x}}</math></p> <p>ii- <math>f = \frac{\pi}{2} \log x  + \frac{a}{x} - \frac{a^3}{9x^3}</math></p>	4	
b)	Write a Program for determination of the factorial value of any integer number	8	
c)	<p>Show how to read the following Numbers from a data file "DATA.DAT", then show how to write in a result file "PROG.RES" according to your format:</p> <p>1.25, 12.348, 15.9, 168, 15, 1254</p>	6	
3.a)	<p>The solution of the quadratic equation <math>AX^2 + BX + C = 0</math> is given by:</p> <p><math>X_{1,2} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}</math></p> <p>i- Draw a flow chart for a program to execute the solution of the above equation.</p> <p>ii- Write a FORTRAN program to execute the above solution and show how to repeat the program for other input data.</p>	10	
b)	<p>i- Write logical expression corresponding to: IF (.NOT. (X .LE. 12.0)) R = X + 31.0</p> <p>ii. Determine the correct format expression, and correct the wrong from the following:</p> <p>1. 100   FORMAT(10X,13F6.2)</p> <p>2. 200   FORMAT(4F7.2,4E13.8)</p> <p>3. 300   FORMAT(7I2,6F5.3,3E12.5)</p>	8	

## Quantum Mechanics II

4.a)	For spin corresponding to $s = 1/2$ , what are the eigenvectors of $\hat{S}_x$ , $\hat{S}_y$ and $\hat{S}_z$ ?	6
b)	Consider the motion of a spinning, but fixed electron which is in a constant uniform magnetic field that points in the $z$ -direction. Suppose that the electron is initially in the state $\alpha_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ . Calculate the eigen-states and eigen-energies of this same system.	12
5.a)	A system in an initial state $\ell$ of unperturbed Hamiltonian $\hat{H}_0(\underline{r})$ affected by a perturbed Hamiltonian $\hat{H}_1(\underline{r}, t) = G(\underline{r})f(t)$ . What is the probability after time ( $t$ ) of the transition to another state $k$ of $\hat{H}_0(\underline{r})$ ?	9
b)	A particle of mass ( $m$ ) confined to a one-dimensional box of width ( $L$ ) has eigen-states $\varphi_n = \frac{1}{\sqrt{L}} e^{i(kx - \omega_n t)}$ and eigen-energies $E_n = \frac{\pi^2 \hbar^2}{2mL^2} (n+1)^2$ , $n = 0, 1, 2, \dots$ . The system is exposed at time $t = -\infty$ when it was in its ground state to a perturbed Hamiltonian $H_1(x, t) = C \frac{\hat{p}_x^2}{2m} \frac{e^{-t^2/\tau^2}}{\tau\sqrt{\pi}}$ , where $C$ and $\tau$ are constants. What state does the perturbation leave the system in at $t = +\infty$ ? [Hint: use $\xi = t/\tau$ ; $\int_{-\infty}^{\infty} d\xi e^{ic\xi - \xi^2} = \sqrt{\pi} e^{-c^2/4}$ ]	9
6.a)	Derive the relation of the scattering cross-section in terms of the scattering amplitude.	7
b)	A low-energy beam of point particles of mass ( $m$ ) and energy ( $E$ ) is scattered from a finite attractive well of depth ( $V_0$ ) and radius ( $R$ ). Calculate the scattering amplitude $f(\theta) = e^{i\delta_0} \sin(\delta_0)/k$ and total cross-section $\sigma$ of this system, where $\delta_0$ is the S-wave phase shift and $k = \sqrt{2mE/\hbar^2}$ .	11

**With our Best Regards**

<b>Examiners:</b>	<i>Prof. Magdy Tadros (*)</i>	<i>Prof. Essam M. Abulwafa (*)</i>
	<i>Dr. Maisa Ismael</i>	



Mansoura University, Faculty of Science, Mathematics Department

Final Exam - Term 1, January 2012.

Final year students (Statistics and Computer Science)

علم الحاسبات (3) - لغة الحاسب - الورقة الثانية

Examiner: Prof. Dr Moawwad El-Mikkawy

Time Allowed: 3 hours

**Answer three questions . All questions carry equal marks**

**Question 1:**

**1-A) TRUE / FALSE**

Circle **T** if the statement is true or **F** if the statement is false.

1. Java is an object-oriented programming language.
2. JVM is a shortcut for Java Virtual Machine.
3. Compiled Java code is bytecode.
4. If  $x = 5$ ,  $y = 10$  and  $z = 4$ , the expression  $(x < y) \&\& (y = z)$  will return a value of false.
5. In Java, the identifiers  $A5$  and  $a5$  refer to the same variable.
6. An identifier in Java must start with a letter, an underscore, or a dollar sign.

**1-B) Fill in the Blanks**

Complete the following sentences.

1. The ..... keyword is used in Java to declare a variable as a constant.
2. The command we use to compile Java code is .....
3. The double ampersand ( $\&\&$ ) is the symbol used in Java to represent the ..... logical operator.
4. Java has eight ..... that are built into the language.
5. When writing nested if statements, not using, or, the improper use, of braces can easily lead to ..... errors.
6. A do / while loop in Java is similar to a while loop, except that .....
7. The ..... statement in Java aborts current iteration of loop and goes to the next iteration of the loop.
8. .... appears in a .class file.
9. .... breaks the loop whenever it is called.
10. The value in a switch can be of type .....

**1-C) MULTIPLE CHOICE**

Select the best response for the following statements.

1. Which of the following symbols is used in Java to represent the AND operator?

- a.  $\&\&$ .
- b.  $\%\%$ .
- c.  $\?\?$ .
- d.  $\|\|$ .

2. Which Java statement can be used as an alternative to using extended ifs?

- a. switch.
- b. level.
- c. for.
- d. when.

3. Which of the following is most frequently used to determine the possible results of expressions containing logical operators?
- logic table.
  - expression simulator.
  - truth table.
  - expression generator.
4. Which term below is used to describe a program that can handle invalid inputs without crashing and does not produce meaningless results?
- iron-clad..
  - foolproof.
  - robust.
  - stout.
5. Which of the following operators has the LOWEST (will be evaluated after all other operators) order of precedence?
- ||.
  - &&.
  - >=.
  - !.

1-D) If b1 and b2 are of type boolean , b1=false and b2=true.What is the value of the following expressions ?

- b1 && b2
- b1 || b2
- b2 || b1
- ! b1
- (! b1) && b2

### Question 2:

2-A) Study the following do / while loop. How many times will it execute? How will the output look?

```
int x = 5 ;
do
{
    x = x + 2;
    System.out.print (x + " ");
} while (x < 30 ) ;
```

2-B) What is the final value of x after executing the following Java code?

```
int x= 0;
for (int i=3 ; i<= 5 ; i++)
{
    x += 1;
    for (int j =0 ; j<3, j++)
    {
        x += 1;
    }
}
System.out.println( "x = " +x);
```

2-C) What does the following code output?

```
for (int i=0; i < 6 ; i++)
{
    for (int j=i ; j>= 0 ; j--)
        System.out.print (j+" ");
    System.out.println();
}
```

2-D) What is the output for the following segment of the Java code?

```
for (int a=1,b=20 ; a < b ; a = a+2, b= b-2)
{
    System.out.println( " a = " +a + " and b = " +b);
}
```

2-E) What is the output for the following segment of the Java code?

```
for (int i=0;i <3 ; i++)
{
    System.out.print (" Pass"+i+":");
    for (int j = 0; j<100; j++)
    {
        if (j == 10) break;
        System.out.print (" j" + " ");
    }
    System.out.println();
}
System.out.println(" Loop Complete.");
```

2-F) What is the following Java code print out?

```
int f = 0 , g = 1;
for (int i = 0; i <= 15 ; i++)
{
    System.out.println(f);
    f = f + g ;
    g = f - g ;
}
```

### Question 3:

3-A) : Multiple Choice / Fill in the Blank :

- In Pascal, the code in a procedure is only executed when the procedure is
  - called
  - declared
  - compiled
- In Pascal, "assignment" of a variable is the name given to
  - specifying a storage location for a variable
  - storing a value in that variable
  - declaring the type of a variable
- In Pascal, if you want the variable called "hits" to take on values which are whole numbers (no fractional part) it should be declared as type .....
- In a Pascal program, if you want a program statement to be ignored or otherwise have no effect upon execution of the program, you can
  - precede the statement with the word "ignore"
  - put the statement inside double quotation marks
  - put the statement inside single quotation marks
  - make it into a comment by enclosing it with "{ }"
- In Pascal, the "case" statement is another form of what statement?
  - while
  - repeat
  - for
  - if
- In computer programming, the sequence of instructions that solves a problem or task is called an .....

7. Which of the following is NOT true about comments in a Pascal program?
- they are used to help humans understand the program
  - they help Pascal discover semantic errors in a program
  - they are used to help humans debug a program
  - they are helpful in allowing others to extend or maintain a program

8. In Pascal, if you want the variable called "batting\_average" to assume values to parts in one thousand (e.g. 0.409) then it should be declared as type .....

9. If you want the variable called "won" to be either true or false, it should be declared as type .....

10. How many times will the following loops execute? (Assume count is an integer)

a) for count := 6 downto 0 do  
writeln('hello');

b) count := 0;  
while count >= 0 do  
begin  
count := count - 2;  
writeln('hello');

3-A) Express the following relationships, using Pascal:

(i)  $a > b > c$  (ii)  $i = j = k$

3-B) Mention six standard functions in Pascal with all details about their arguments.

3-C) Write a Pascal program to read the value of a positive integer n as an input data and then

compute  $\sum_{r=1}^n \frac{(-1)^{r+1}}{r}$ .

3-D) Write a Pascal program to read the elements of a matrix  $A = (a_{i,j})_{n \times n}$  and then compute the sum of all elements of A above the main diagonal.

#### Question 4:

4-A) What are the main differences between while / do and repeat / until loops in Pascal?

4-B) If the value of mark is 67, what will be printed after the execution of the following Pascal code?

```
case mark div 10 of
  7,8,9,10 : write (' Very Good');
  6        : write(' Good');
  5        : write (' Fair');
  4        : write('Poor');
  0,1,2,3  : write(' Fail')
```

end

4-C) Write a Pascal function subprogram Big to evaluate the smallest value for three given integers.

4-D) In the game of buzz-fizz each player adds one to the previous player's number and calls out the number unless it divides exactly by three or five. If it divides by three he calls out 'buzz' instead and if it divides by five he calls out 'fizz'. If it divides by both three and five he calls out 'buzz fizz'.  
construct a piece of Pascal code to output the numbers one to one hundred, in figures, substituting 'buzz', 'fizz', or 'buzz fizz', if appropriate.

= = = = = **END OF EXAM** = = = = =

Kind regards,  
The examiner  
Page 4/4

Mansoura University

29-12-2012

Faculty of science

Course: OR

Code:R421

Mathematics Department

(4<sup>th</sup> level exam)

Time:2Hours

Answer the following questions

No. of Questions:4

Total Mark:80

Question:1

(20 marks)

- (a) If it is possible solve the following mathematical models by using the graphical method.

<p>Maximize <math>Z = 4x_1 + 5x_2,</math></p> <p>(i) subject to <math>x_1 + x_2 \geq 5,</math></p> <p><math>2x_1 + 4x_2 \leq 16,</math></p> <p><math>x_1, x_2 \geq 0.</math></p>	<p>Minimize <math>Z = 4x_1 + 3x_2,</math></p> <p>(ii) subject to <math>-x_1 + x_2 \geq 3,</math></p> <p><math>x_1 + x_2 \leq 8,</math></p> <p><math>x_1, x_2 \geq 0.</math></p>
--	---

Maximize  $Z = x_1 + 3x_2,$

subject to  $-x_1 + 3x_2 \leq 9,$

(iii)  $x_1 + x_2 \leq 6,$

$x_1 - x_2 \geq 2,$

$x_1, x_2 \geq 0.$

- (b) Express the following **L.P** in the standard (matrix) form:

Maximize  $Z = 4x_1 + 2x_2 + 6x_3,$

subject to  $2x_1 + 3x_2 + 2x_3 \geq 5,$

$3x_1 + 4x_2 = 8,$

$6x_1 - 4x_2 + x_3 \leq 10,$

$x_1, x_2, x_3 \geq 0.$

Question:2

(20 marks)

- (a) Use **The big M-method** to solve

Maximize  $Z = 3x_1 + 2x_2,$

subject to  $2x_1 + x_2 \leq 1,$

$3x_1 + 4x_2 \geq 4,$

$x_1, x_2 \geq 0.$



(b) Construct the dual to the primal problem:

$$\begin{aligned}
 &\text{Maximize} && Z = 3x_1 + 10x_2 + 2x_3, \\
 &\text{subject to} && 3x_1 + 4x_2 + 2x_3 \leq 7, \\
 & && 3x_1 - x_2 + 4x_3 = 6, \\
 & && 6x_1 - 4x_2 + 2x_3 \geq 10, \\
 & && x_1, x_2, x_3 \geq 0.
 \end{aligned}$$

Question:3

(20 marks)

(a) Find the initial feasible solution to the following transportation problem by:

- (i) *north-west corner rule,*  
(ii) *Minimum cost rule,*

		<i>To</i>				
		1	2	3	4	
						<i>Supply</i>
<i>From</i>	1	4	3	10	7	15
	2	1	0	6	2	25
	3	5	8	15	9	10
		<i>Demand</i>	15	10	20	5

(b) By using *Vogel's approximation method* solve the above problem.

Question:4

(20 marks)

Solve the following *Assignment* problem:

	I	II	III	IV
1	10	5	9	18
2	13	9	6	12
3	3	2	4	4
4	18	9	12	17

WITH THE BEST WISHES



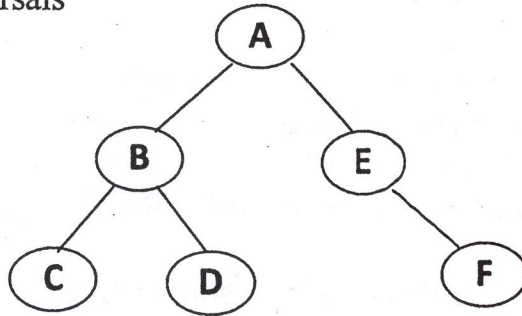
Please answer the following questions:

**Q1: Write short notes for the following:**

1. Removing doubly linked list Node, draw an example.
2. Operations on Linked Lists.
3. Stack applications
4. Queues operations
5. Tree terminology
6. Linear collections and Nonlinear collections
7. Multidimensional arrays and Jagged arrays, give an example.
8. Algorithms and programs.
9. Linked Lists and Array Lists.
10. Sequential search and Binary search, give an example.

**Q2: For the following tree, show the sequence of processing nodes for:**

- a. Depth-first traversals (Preorder, inorder, and postorder)
- b. Breadth-first traversals



**And then find the expression trees for the following:**

c.  $A + B * [C * D + E * (F + G)]$

d. 
$$\frac{A - B * (C * (D - E))}{F + G * H}$$

**Q3: Show step by step how the following array can be sorted using:**

72	54	59	30	31	78	2	77	82	72
----	----	----	----	----	----	---	----	----	----

- a. Bubble sort algorithm.
- b. Insertion sort algorithm.



**Q4: Explain the following codes:**

A.

```
Public Class Node
    Public Element As Object
    Public Link As Node
    Public Sub New()
        Element = Nothing
        Link = Nothing
    End Sub
    Public Sub New(theElement As Object)
        Element = theElement
        Link = nothing
    End Sub
End Class
```

B.

```
Module Module1
    Sub Main()
        Dim theArray As New CArray(11)
        Dim index As Integer
        For index = 0 To 11
            theArray.Insert(Int(100 * Rnd() + 1))
        Next
        Console.WriteLine("Before sorting: ")
        theArray.showArray()
        Console.WriteLine("During sorting: ")
        theArray.BubbleSort()
        'theArray.SelectionSort()
        'theArray.InsertionSort()
        Console.WriteLine("After sorting: ")
        theArray.showArray()
        Console.Read()
    End Sub
End Module
```

*Good luck*

*Dr. Abdelhameed Faray (1/2013)*

Mansoura university  
Faculty of science  
Math. Depart

1<sup>st</sup> term  
2011/2012  
4<sup>th</sup> year

المقرر: إحصاء رياضي (عمليات عشوائية)  
الزمن : 3 ساعات  
التاريخ : 2012/1/22

**Answer the following questions**

**Q1: ( 13.5 marks)**

(a) Let  $\{X_n, n = 0, 1, 2, \dots\}$  is a Markov chain with state space  $S = \{0, 1, 2\}$  has a transition

probability matrix  $P = \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.8 & 0.1 & 0.1 \\ 0.3 & 0.6 & 0.1 \end{bmatrix}$  and starting vector  $(0.4, 0.3, 0.3)$ . Find

- (i)  $P(X_2 = 1)$  (ii)  $P(X_0 = 0, X_1 = 1, X_2 = 1)$  (iii)  $P(X_5 = 2 | X_3 = 1)$

**Q2: ( 13.5 marks)**

(b) Let  $\{X_n, n \geq 0\}$  be a Markov chain with state space  $S = \{1, 2, 3\}$  and transition

probability matrix  $P = \begin{bmatrix} 0.3 & 0.5 & 0.2 \\ 0.6 & 0 & 0.4 \\ 0 & 0.4 & 0.6 \end{bmatrix}$ . Find the ergodic probabilities of the

states if it exist.

**Q3: ( 13.5 marks)**

(a) Let  $\{X_n, n = 0, 1, 2, \dots\}$  is a Markov chain. Prove the Chapman-Kolmogorov equation

$$P_{ij}^{(n+m)} = \sum_{k=0}^{\infty} P_{ik}^n P_{kj}^m$$

(b) Let  $\{N(t), t \geq 0\}$  be a Poisson process with rate  $\lambda$ . Show that  $E \left[ \left| \frac{N(t)}{t} - \lambda \right|^2 \right] \rightarrow \infty$  as  $t \rightarrow \infty$ .

**Q4: ( 13.5 marks)**

Three coins are placed in a row on a table. At each stage, a coin is selected at random and turned over. Let  $\{X_n, n = 1, 2, \dots\}$  where  $X_n$  is the number of heads

after the  $n$ -th trial is a Markov chain with t. p.m  $P = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1/4 & 0 & 3/4 & 0 \\ 0 & 1/2 & 0 & 1/2 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ . If we

started with 2 heads, then find the probability that

- (a) There are 3 heads after the 1<sub>st</sub> trial (b) There is one head after 2<sub>nd</sub> trial.

**Q5: ( 13.5 marks)**

- (a) State two definitions for Poisson process and prove that they are equivalent.  
 (b) Suppose that the customers arrive at a bank according a Poisson process with rate 0.2 per minute. Each customer arriving at a bank has probability  $\frac{1}{3}$  of being recorded. Find  
 (i) The probability that the number of customers are recorded is 5 in the time interval 9 Am to 10 Am.  
 (ii) The mean of the number of recorded customers in the time interval 9 Am to 11 Am.