/٤١٧ - تحليل مركب (٢)

المستوى الرابع البرنامج: الرياضيات إسم المقرر: ر٤١٧ تحليل مركب (٢) (خ)	كلية العلوم – قسم الرياضيات	الفصل الدراسى الثانى ٢٠١٣ الزمن: ساعتان التاريخ:٢٠١٣/٥/٢٨

Answer the following questions:

1. a. Prove that $\int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$ (10 marks)

   b. Prove that $\int_0^{2\pi} \frac{d\theta}{1 + a\sin\theta} = \frac{2\pi}{\sqrt{1-a^2}} \quad (|a| < 1)$ (10 marks)

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2. a. Define pole of order m at $z = z_0$. If f(z) has a pole of order m at $z = z_0$. Prove that

$$\operatorname{Res}[f, z_0] = \lim_{z \to z_0} \frac{1}{(m-1)!} \frac{d^{(m-1)}}{dz^{(m-1)}} [(z - z_0)^m f(z)]$$ (10 marks)

   b. Find Laurent expansion for $f(z) = \frac{z + \cos z}{z^3}$ about $z = 0$. (5 marks)

   c. If $w = f(z)$ is analytic function. Then prove that $\frac{\partial(u,v)}{\partial(x,y)} = |f'(z)|^2$. (5 marks)

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3. a. Find the bilinear transformation that transforms $z = 0, 1, -2$ to $w = \infty, 1, i$, respectively. (10 marks)

   b. Prove under $w = \frac{1}{z}$ straight lines and circles are mapped onto straight lines or circles. (5 marks)

   c. Find the image of $x + 2y - 1 = 0$ and $x - 2y + 1 = 0$ under $w = \frac{1}{z}$. (5 marks)

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4. a. Prove that $f(z) = \cos z$ not bounded. Find the image of $\frac{-\pi}{3} \le x \le \frac{\pi}{3}$ and $y = 0$ under $w = \cos z$ (8 marks)

   b. Describe a Riemann surface for $w = z^{\frac{1}{3}}$. (8 marks)

   c. Define analytic continuation of $w = f(z)$. (4 marks)

مع تمنياتنا بالنجاح والتوفيق

الممتحن: أ.د./ محمد كمال عبد السلام عوف

المستوى الرابع ٤١٢/

الشعبة: إحصاءوحاسب + رياضيات المادة: تحليل دالي	كلية العلوم – قسم الرياضيات	دور مايو ٢٠١٣ الزمن: ساعتـــــــان التاريخ: ٢٠١٣/٥/٢٥

Answer the following questions

First : Objective questions : **(20 marks)**

Among the following statements mark the true and false ones with (√) and (×) respectively. Justify your answer for ONLY TWO of them :

(i) In a metric space, a set $A \neq \Phi$ is closed if and only if $\overline{A}$ is closed. ....( )

(ii) For any sequences $(\alpha_n),(\beta_n) \in \ell^2$ we have

$$\sum_{n=1}^{\infty} |\alpha_n \beta_n| \leq \left(\sum_{n=1}^{\infty} |\alpha_n|^2\right)^{1/2} \cdot \left(\sum_{n=1}^{\infty} |\beta_n|^2\right)^{1/2} \text{ . ...( )}$$

(iii) .The sequence ($(-1)^n$) belongs to the space ℓ^∞. ....( )

(iv) Any subspace of a Banach space is also a Banach space. ....( )

(v) If $A = \{x_1, x_2, x_3, x_4, x_5, x_6\} \subseteq \ell^\infty$, it follows that the linear hull H(A) is separable. ....( )

(vi) For any normed spaces E,F over K, the space L(E,F) is a Banach space. ....( )

(vii) Any set $U \neq \Phi$ can be converted to a metric space. ....( )

(viii) Any two linearly homeomorphic normed spaces are linearly isometric. ....( )

(ix) Every linear operator $T : R \to R$ is continuous on R. ....( )

(x) The space ℓ^4 is linearly homeomorphic to the space K^4 . ...( )

Second : Subjective questions **(20 marks each)**

[1] a. Define : a metric space – the space ℓ^p. [4 marks]

Show that , for p > 1, and for all $(\alpha_n), (\beta_n) \in \ell^p$, the sequence $(\alpha_n+\beta_n)$ belongs to ℓ^p , and

$$\left(\sum_{n=1}^{\infty} |\alpha_n + \beta_n|^p\right)^{1/p} \leq \left(\sum_{n=1}^{\infty} |\alpha_n|^p\right)^{1/p} + \left(\sum_{n=1}^{\infty} |\beta_n|^p\right)^{1/p} .$$ [10 marks]

b. Let X be a non-empty set. A mapping $L: X \times X \to R$ satisfies the following conditions :

(i) $L(x,y) = 0$ if and only if $x = y$, and (ii) $L(x,z) \leq L(y,x) + L(y,z)$ $\forall\, x,y,z \in X$.

Prove that L is a metric on X. [6 marks]

[2] a. Define : a separable space. [2 marks]

Show that,for p > 1, the space ℓ^p is separable . [8 marks]

b. Let E,F be normed spaces over K , $E \neq \{0\}$, and let $T : E \to F$ be a linear mapping of E onto F. Prove that T is one-to-one and T^{-1} is bounded if and only if there is a constant m > 0 such that $\|Tx\| \geq m$ for all $x \in E$ with $\|x\| = 1$. [10 marks]

[3] a. If $T : R^3 \to R^3$ is defined by $T(\alpha,\beta,\gamma) = (\alpha-\gamma, 2\alpha+\gamma, 3\beta)$; show that T is a bounded linear operator on R^3, and then find $\|T\|$. [9 marks]

b. Show that if E is a finite-dimensional normed space, then E is a Banach space, and every linear transformation $T : E \to G$ is bounded. (G; being any normed space) [11 marks]

Best Wishes

المستوى الرابع - الرياضيات - ميكانيكا متقدمة ر٤٢٤

كلية العلوم قسم الرياضيات المستوى: الرابع شعبة:الرياضيات الدرجة الكلية: ٨٠ درجة	Mansoura University	دور مايو: ٢٠١٣ الزمن: ساعتان المادة: ميكانيكا متقدمة كود المادة:ر ٤٢٤ التاريخ: ٢٠١٣/٦/١٥

أجب عن الأسئلة الأتية

السؤال الأول: [20 درجة]

١) أذكرمع البرهان تفسير بوانسو الهندسى لحركة الجسم المتماسك فى حالة أويلر. [10درجة]

٢) إختزل إلى الصورة القياسية التكامل $z=\int_0^x \frac{dt}{\sqrt{\cos t}}$ ثم أوجد x بدلالة z كدالة ناقصية مقياسها $k<1$. [10درجة]

السؤال الثانى: [20 درجة]

١) إستخدم تكامل المساحة ومعادلات بواسون للتعبير عن متجه السرعة الزاوية بدلالة $\boldsymbol{\gamma}$ وتفاضله بالنسبة للزمن على الصورة $\boldsymbol{\omega}=\frac{f\,\boldsymbol{\gamma}+\dot{\boldsymbol{\gamma}}\times\boldsymbol{\gamma}\mathbf{I}}{\boldsymbol{\gamma}\mathbf{I}\cdot\boldsymbol{\gamma}}$ حيث f ثابت إختيارى. [10درجة]

٢) أثبت أن الدوران المنتظم للجيروسكوب الواقف يظل غير مستقر إذا كان $|r_0|<\frac{2\sqrt{mgAz_0}}{C}$ بينما يصبح مستقرا إذا كان $|r_0|>\frac{2\sqrt{mgAz_0}}{C}$ حيث r_0 هى السرعة الزاوية للدوران وأن A,A,C هى عزوم القصور الرئيسية و z_0 هو بعد مركز ثقله عن النقطة الثابتة. [10درجة]

السؤال الثالث: [20 درجة]

١) أذكر الحالات القابلة للتكامل لجسم متماسك مثبت من نقطة ويتحرك تحت تأثير وزنه فقط. [3درجة]

٢) إستنتج التكامل الأول الرابع فى حالة كوفالفسكايا. [10درجة]

٣) أوجد المحل الهندسى للمحاور المارة بالنقطة O والتى تتساوى عزوم القصور الذاتى حولها لجسم متماسك إختيارى و متماثل محوريا [7 درجة]

السؤال الرابع: [20 درجة]

١) عرف الجيروسكوب- الجيروستات ثم إستنتج معادلات حركة الجيروستات وذلك بإعتبار سرعة دوران الجيروسكوب ثابتة . [10درجة]

٢) نحلة منتظمة كتلتها m وتتحرك حول نقطة ثابته فيها O على محور تماثلها وكان بعد مركز ثقلها عن O يساوى l و عزوم القصور الرئيسية هى A,A,C فإذا بدأت النحلة حركتها عندما كان محورها رأسيا إلى أعلى والسرعة الزاوية للنحلة حول محورها n. أثبت أنه عندما يكون $n^2C^2=3mglA$ فإن أقصى ميل لمحور النحلة على الرأسى هو $\frac{\pi}{3}$ [10درجة]

مع أطيب الأمنيات بالنجاح و التوفيق

أ.د/حمد حلمى يحيى

المستوى الثالث - رياضيات احصاء وحاسبات - دوال خاصة ، ٣٢٤

الفرقة: الثالثة المادة : دوال خاصة كود المادة: 324 الدرجة الكلية:80 درجة	كلية العلوم – قسم الرياضيات برنامج: الرياضيات– الاحصاء وعلوم الحاسب	دور مايو 2013 الزمن: ساعتين التاريخ : 2013/6/13

اجب عن الاسئلة الاتية

السؤال الاول: (20 درجة)

احسب التكاملات الاتية

(ا)- $\int_0^\infty e^{-t^2}\,dt$ (6 درجات)

(ب)- $\int_0^\infty \frac{1}{1+y^4}\,dy$ (6 درجات)

(ج)- اثبت ان $\Gamma(2x) = \dfrac{2^{2x-1}\Gamma(x)\Gamma(x+\frac{1}{2})}{\sqrt{\pi}}$ (8 درجات)

السؤال الثانى : (20 درجة)

(ا)- اثبت ان ${}_2F_1(\frac{1}{2},1,\frac{3}{2},z^2) = \frac{1}{2z}\log\frac{1+z}{1-z}$ **(7 درجات)**

(ب)- اوجد $P_3(x)$ حيث $P_0(x)=1$ و $P_1(x)=x$ ($P_n(x)$ هى كثيرة حدود ليجندر) **(5 درجات)**

(ج)- اوجد مفكوك الدالة $f(x)=\begin{cases}1 & 0\le x<1\\ 0 & -1\le x<0\end{cases}$

فى الصورة $\sum_{m=0}^{\infty} A_m P_m(x)$ **(8 درجات)**

السؤال الثالث: (20 درجة)

(ا)- اثبت انة عندما n عدد صحيح فان $J_{-n}(x) = (-1)^n J_n(x)$ **(7 درجات)**

(ب)- اوجد قيمة $\int J_1(\sqrt[3]{x})\,dx$ **(6 درجات)**

(ج)- اثبت ان $4J_0''' + 3J_0'' + 3J_3 = 0$ **(7 درجات)**

السؤال الرابع: (20 درجة)

1. **اذكر صورة ردريجيوس لكثيرات حدود لاجير $L_n(x)$ ومن ثم اوجد $L_2(x)$** **(6 درجات)**
2. **اذكر خواص التعامد لكثيرات حدود لاجير** **(4 درجات)**
3. اثبت ان $L_n(x) = n!\,{}_1F_1(-n,1,x)$ **(10 درجات)**

مع اطيب التمنيات بالتوفيق والنجاح

د. عبد المنعم لاشين

Mathematics Department Date : 11-6-2013 Full Mark : 80	Faculty of Science	4^{th} Final Exam. Mathematics group Partial Differential equations R 429

المستوى ٤ ع
رياضيات
معادلات تفاضلية
جزئية
٤٢٩ ر

Answer the following questions , each question 20 marks

[1]a) Show that the Dirichlet problem $\nabla^2 u = 0$ in R, $u|_S = f$ has a unique solution, where R is a region in the xy-plane , S is the boundary of R

b) Find D' Alembert's solution of the wave equation $\dfrac{\partial^2 u}{\partial x^2} = \dfrac{1}{c^2}\dfrac{\partial^2 u}{\partial t^2}$ subject to the Cauchy initial conditions u(x,0)=f(x) and $\left(\dfrac{\partial u}{\partial t}\right)_{t=0} = g(x)$ and show that the problem is well – posed.

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[2] Show that the solution of the equation $x^2\dfrac{\partial^2 u}{\partial x^2} = \dfrac{1}{c^2}\dfrac{\partial^2 u}{\partial t^2}$, c is a const. which satisfies the boundary conditions u(a, t)=u(2a,t)=0 is

$$u(x,t) = \sin\left[\lambda \ln\left(\frac{x}{a}\right)\right]\left(\frac{x}{a}\right)^{1/2}(A\cos\omega t + B\sin\omega t).$$

Where $\omega^2 = c^2(\lambda^2 + 1/4)$, $\lambda = n\pi/\ell n2$, n is a positive integer.

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[3] Find the solution of the heat conduction equation $\nabla^2 u = \dfrac{1}{k}\dfrac{\partial u}{\partial t}$ in R, where R is a circular region of radius a subject to the boundary conditions u(a,t)=0 , $t \ge 0$ and $u(r,0) = f(r)$,$0 \le r < a$

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[4]a) Show that the solution of Laplace's equation $\nabla^2 u = 0$ in the region $x > 0$, $0 < y < a$ satisfying u(x,0)=f(x) and u(x,a) = 0 where f(x) is a given function , and a is a constant is

$$u(x,y) = \frac{1}{\pi}\int_0^\infty\left[\int_{-\infty}^{\infty}\frac{\sinh\lambda(a-y)}{\sinh\lambda a}f(\xi)\cos\lambda(\xi - x)d\xi\right]d\lambda.$$

b) By using Laplace's transform method, solve the equation

$\dfrac{\partial u}{\partial t} + x\dfrac{\partial u}{\partial x} = x$, $(x > 0\ , t > 0)$ **where u=u(x,t), given the boundary conditions u(x,0)=0 for x > 0 and u(0,t)=0 for t > 0.**

د.محاسن موسى

مع أطيب الأمنيات بالنجاح والتوفيق إن شاء الله

المستوى الرابع - الاحصاء وعلوم الحاسب - نمذجة ومحاكاة / ٤٢٦
ار رياضيات

Mansoura University
Faculty of Science
Mathematics Department

Final exam
Second term
May 2013

4th level students (Mathematics / Statistics and computer Science programme)

Subject: Math 426 (Modelling and Simulations)

Date: 08 /06/2013 **Time allowed: Two hours**

Answer the following questions: **Total marks: 80**

Question one:

A) Write down and find the equilibria for each of the following difference equation models: the Beverton-Holt model, the Ricker model, and the Nicholson-Bailey model. **(10 marks)**

B) Construct a mathematical model to describe the competition between two species. Explain your model, find the steady states and discuss their stability. **(10 marks)**

Question two:

A) Write down a discrete-time model for measles with vaccination. **(5 marks)**

B) Construct an SIS model for an infection spreading in a closed population with **varying** size. Explain your model, find the steady states and discuss their stability. **(15 marks)**

Question three:

A) Write down both the von Bertalanffy and Gompertz models for the tumour growth. **(5 marks)**

B) Prove that in a large **spherical** tumour there is a shell of proliferating cells, whose thickness depends on the excess nutrient concentration above a threshold $(\mathbf{c_2 - c_1})$, how fast the nutrient is consumed $\mathbf{k}$ and how fast it diffuses $\mathbf{D}$, but not on the size of the tumour itself. **(15 marks)**

Question four: **(20 marks)**

Construct an SIRS model for an infection spreading in a closed population with constant size and study the possibility to vaccinate the susceptible individuals with rate ψ. Find the critical age above which susceptible individuals should be vaccinated to protect the population from the infection.

Best regards,
The examiner
Dr. Muntaser Safan

المستوى الرابع- رياضيات - هيدروديناميكا / ٤٥

جامعة المنصورة

كلية العلوم

قسم الرياضيات

المستوى الرابع رياضيات

هيدروديناميكا

دور مايو 2013

الزمن: ساعتان

Answer the Following Questions:

1) a- State, with proof, the continuity equation.

b- Show that $\psi = U \sin\theta (r - \frac{R^2}{r})$ represents the stream function a 2-dimensional flow of an incompressible irrotational flow and find the corresponding velocity potential.

2) a- Deduce the Bernoulli's theorem for unsteady irrotational flow under a conservative force field.

b- The radius of a sphere immersed in an infinite ocean of liquid varies according to the relation r = A + a cos nt; A, a , n constants. If the velocity potential has the form $\varphi = \frac{(f(t))}{r}$, find f(t) and the maximum pressure attained on the sphere assuming that $A \leq 5a$.

3) Two sources each of strength m are placed at the points (-a, 0) and (a, 0) and a sink of strength 2m is placed at the origin, show that the streaming lines are the curves $(x^2 - y^2)^2 - a^2(x^2 - y^2 + 2\lambda x y) = 0$ where λ is a parameter.

4) Use the method of separation of variables to find the velocity potential of a system of a sphere of radius a and uniform stream U. Find the equation of the stream lines and the pressure at any point on the sphere.

د/ محمود محمد عبد الحفيظ

المستوى الرابع - الإحصاء وعلوم الحاسب - نظرية الأشكال ، ٤١٢
الرياضيات

El-Mansoura- Egypt	Fourth level of B.Sc.	المنصورة – مصر
Mansoura University Faculty of Science Mathematics Department **Second Term**	Program: Math. and Statistics & Computer Science Subject: Graph Theory Course Code 412: **Date: 1 June. 2013**	جامعة المنصورة كلية العلوم قسم الرياضيات **Time: 2 hours**

Answer the following ~~five~~ four questions: **Mark**

1- **a-** Find the number of edges $|E(G)|$ of the graph G of each of: **(10 points each item 2 points)**

(i) G is oriented digraph with n vertices and maximal number of arcs.
(ii) Regular graph G of order 1 with 2 n vertices .
(iii) A complete rooted 2-tree with 7 vertices,
(iv) G is a tree with $2n$ vertices ,
(v) G is a simple graph having maximum number of edges, $2n$ vertices and no triangles.

b- Give an example of each of : **(10 points each item 2 points)**

(i) A cubic graph with 6 vertices.
(ii) Two non-isomorphic digraphs with 3 vertices and 3 arcs.
(iii) A graph with n vertices and its diameter = n -1, for each +ve integer n.
(vi). A graph with (girth) $g(G) = n = c(G)$ (circumference) for each +ve integer n .
(v) Two distinct trees with 4 vertices.

2 -a Prove that a graph G is regular of order 2 $\Leftrightarrow$ each component of G is a cycle. **(8 points)**

b - Prove that In any graph the number of vertices of odd degrees is even. **(6 points)**

c - Is there a complete rooted 2-tree with even number of vertices? why? **(6 points)**

3. a- If G is a plane graph with k components , n vertices , m edges and r regions, prove that $n - m + r = 1 + k$. **(10 points)**

b- Show that the complete graph $\mathbf{K}_5$ is nonplanar and then prove that $\mathbf{K}_n$ is a nonplanar graph for each $n \geq 5$. **(10 points)**
Also, show that the complete graph K_n is a nonbipartite graph for each $n \geq 3$.

4 – a. Let G be a simple graph with n vertices and k components, show that $n - k \leq |E(G)| \leq \binom{n-k+1}{2}$. And then give an example of a simple graph with 10 vertices and 2 components having : **(10 points)**
(i) minimal number of $|E(G)|$, and **(ii)** maximal number of $|E(G)|$.

b - Let T_1 and T_2 be two trees with the same number of vertices n. **(10 points)**
Prove or disprove that
(i) Both T_1 and T_2 have the same number of edges.
(ii) Give two non-isomorphic trees T_1 and T_2 both have 5 vertices.
(iii) If e is an edge and v is a vertex in T_1 , then $T_1 - e$ and $T_1 - v$ are trees.?

Good luck **Total (80 points)**