

Mansoura university 2nd term المقرر: تحليل التباين (ر ٤٣٤)
 Faculty of science 2012/2013 الزمن : ساعتان
 Math. Depart 4th year (stat., computer sci.) التاريخ : ٢٠١٣/٥/٢٨
 Final exam

Answer the following questions

Q1: (27 marks)

For the observations in the following table

treatments	observations					
A	7	6	8	5	9	7
B	8	9	10	7	8	6
C	7	8	10	5	6	3
D	8	6	5	4	9	4

- (a) Test the hypothesis that $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$ at level of significance $\alpha = 0.05$ where $f_{0.05}(3,20) = 3.10$
 (b) Use Bartlett's test to test the homogeneity of variances at level of significance $\alpha = 0.01$ where $b_4(0.01,6) = 0.5430$,

Q2: 27 marks)

- (a) Construct one way analysis of variance table and test the hypothesis $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$, at level of significance $\alpha = 0.05$, for the following observations

Treatment	Observations					
A	7	10	9	8	10	10
B	5	6	4	8	7	6
C	8	9	10	6	3	6
D	5	6	4	3	7	5

Where $f_{0.05}(3,20) = 3.10$

- (b) If H_0 is rejected, use Scheffe's test to compare between four population means
 (c) Compare between the two groups of treatments (A, B, C) versus (D) using the contrast $w = \mu_1 + \mu_2 + \mu_3 - 3\mu_4$, where $f_{0.05}(1, 20) = 4.35$.

Q3: (26 marks)

Suppose that we are interested in the yields of 3 varieties A, B and C of wheat using 4 different fertilizers, planted in 12 randomly selected pieces of land with the same fertility assumption, production was, as in the following table. The yields for the 3 varieties of wheat measured in 100 kg

	Fertilizers			
Wheat	I	II	III	IV
A	70	56	51	61
B	66	60	55	60
C	77	67	59	65

- (a) Are there any differences between the impact of different types of fertilizers
 (b) Are there any differences between the productions of different types of wheat?
 [Use $\alpha = 0.05$ and $f_{0.05}(2,6) = 5.14$, $f_{0.05}(3,6) = 4.76$].

<p>دور مايو ٢٠١٣ الزمن: ساعة التاريخ: ٢٠١٣/٥/٢٥</p>	 كلية العلوم - قسم الرياضيات	<p>الشعبة: إحصاء وحاسب + رياضيات المادة: تحليل دالي</p>
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Answer the following questions

First : Objective questions :

(20 marks)

Among the following statements mark the true and false ones with (✓) and (×) respectively. Justify your answer for **ONLY TWO** of them :

- (i) In a metric space, a set $A \neq \Phi$ is closed if and only if \bar{A} is closed.()
- (ii) For any sequences $(\alpha_n), (\beta_n) \in \ell^2$ we have
- $$\sum_{n=1}^{\infty} |\alpha_n \beta_n| \leq \left(\sum_{n=1}^{\infty} |\alpha_n|^2 \right)^{1/2} \cdot \left(\sum_{n=1}^{\infty} |\beta_n|^2 \right)^{1/2} . \dots\dots\dots()$$
- (iii) .The sequence $((-1)^n)$ belongs to the space ℓ^∞ ()
- (iv) Any subspace of a Banach space is also a Banach space.()
- (v) If $A = \{x_1, x_2, x_3, x_4, x_5, x_6\} \subseteq \ell^\infty$, it follows that the linear hull $H(A)$ is separable.()
- (vi) For any normed spaces E, F over K , the space $L(E, F)$ is a Banach space.()
- (vii) Any set $U \neq \Phi$ can be converted to a metric space.()
- (viii) Any two linearly homeomorphic normed spaces are linearly isometric.()
- (ix) Every linear operator $T : \mathbb{R} \rightarrow \mathbb{R}$ is continuous on \mathbb{R} ()
- (x) The space ℓ^4 is linearly homeomorphic to the space K^4 ()

Second : Subjective questions

(20 marks each)

- [1] a. Define : a metric space – the space ℓ^p . [4 marks]

Show that , for $p > 1$, and for all $(\alpha_n), (\beta_n) \in \ell^p$, the sequence $(\alpha_n + \beta_n)$ belongs to ℓ^p , and

$$\left(\sum_{n=1}^{\infty} |\alpha_n + \beta_n|^p \right)^{1/p} \leq \left(\sum_{n=1}^{\infty} |\alpha_n|^p \right)^{1/p} + \left(\sum_{n=1}^{\infty} |\beta_n|^p \right)^{1/p} . \quad [10 \text{ marks}]$$

- b. Let X be a non-empty set. A mapping $L: X \times X \rightarrow \mathbb{R}$ satisfies the following conditions :

(i) $L(x, y) = 0$ if and only if $x = y$, and (ii) $L(x, z) \leq L(y, x) + L(y, z) \quad \forall x, y, z \in X$.

Prove that L is a metric on X . [6 marks]

- [2] a. Define : a separable space. [2 marks]

Show that, for $p > 1$, the space ℓ^p is separable . [8 marks]

- b. Let E, F be normed spaces over K , $E \neq \{0\}$, and let $T : E \rightarrow F$ be a linear mapping of E onto F . Prove that T is one-to-one and T^{-1} is bounded if and only if there is a constant $m > 0$ such that $\|Tx\| \geq m$ for all $x \in E$ with $\|x\| = 1$. [10 marks]

- [3] a. If $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is defined by $T(\alpha, \beta, \gamma) = (\alpha - \gamma, 2\alpha + \gamma, 3\beta)$; show that T is a bounded linear operator on \mathbb{R}^3 , and then find $\|T\|$. [9 marks]

- b. Show that if E is a finite-dimensional normed space, then E is a Banach space, and every linear transformation $T : E \rightarrow G$ is bounded. (G ; being any normed space) [11 marks]

✧ Best Wishes ✧

El-Mansoura- Egypt	Fourth level of B.Sc.	المنصورة - مصر
Mansoura University Faculty of Science Mathematics Department Second Term	Program: Math. and Statistics & Computer Science Subject: Graph Theory Course Code 412: Date: 1 June. 2013	جامعة المنصورة كلية العلوم قسم الرياضيات Time: 2 hours

Answer the following ^{four} ~~five~~ questions:

Mark

- 1- a- Find the number of edges $|E(G)|$ of the graph G (10 points each item 2 points)
of each of:
- (i) G is oriented digraph with n vertices and maximal number of arcs.
 - (ii) Regular graph G of order 1 with $2n$ vertices .
 - (iii) A complete rooted 2-tree with 7 vertices,
 - (iv) G is a tree with $2n$ vertices ,
 - (v) G is a simple graph having maximum number of edges, $2n$ vertices and no triangles.
- b- Give an example of each of : (10 points each item 2 points)
- (i) A cubic graph with 6 vertices.
 - (ii) Two non-isomorphic digraphs with 3 vertices and 3 arcs.
 - (iii) A graph with n vertices and its diameter = $n - 1$, for each +ve integer n .
 - (vi). A graph with (girth) $g(G) = n = c(G)$ (circumference) for each +ve integer n .
 - (v) Two distinct trees with 4 vertices.
- 2 -a Prove that a graph G is regular of order 2 \Leftrightarrow each component of G is a cycle. (8 points)
- b - Prove that In any graph the number of vertices of odd degrees is even. (6 points)
- c - Is there a complete rooted 2-tree with even number of vertices? why? (6 points)
3. a- If G is a plane graph with k components , n vertices , (10 points)
 m edges and r regions, prove that $n - m + r = 1 + k$.
- b- Show that the complete graph K_5 is nonplanar and (10 points)
then prove that K_n is a nonplanar graph for each $n \geq 5$.
Also, show that the complete graph K_n is a nonbipartite graph for each $n \geq 3$.
- 4 - a. Let G be a simple graph with n vertices and k components, (10 points)
show that $n - k \leq |E(G)| \leq \binom{n-k+1}{2}$. And then give an example
of a simple graph with 10 vertices and 2 components having :
- (i) minimal number of $|E(G)|$, and (ii) maximal number of $|E(G)|$.
- b - Let T_1 and T_2 be two trees with the same number of vertices n . (10 points)
Prove or disprove that
- (i) Both T_1 and T_2 have the same number of edges.
 - (ii) Give two non-isomorphic trees T_1 and T_2 both have 5 vertices.
 - (iii) If e is an edge and v is a vertex in T_1 , then $T_1 - e$ and $T_1 - v$ are trees.?

Good luck

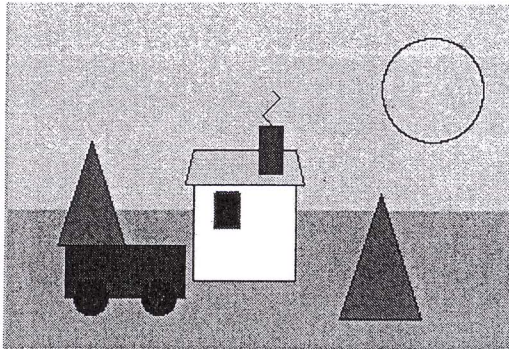
Total (80 points)



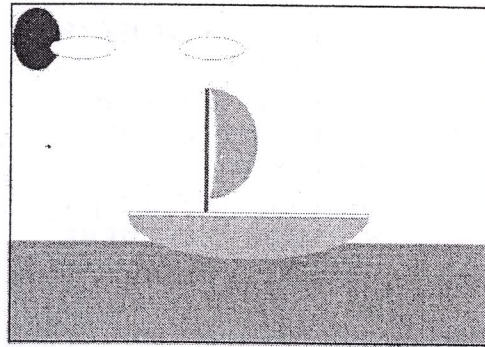
Please answer the following questions:

Q1: Write codes for the following graphs:

(20 Marks)



(a)



(b)

Q2: For each of the following fragments of code, write what the output would be.

(10 Marks)

```
num = 10
while num > 3:
    print num
    num = num - 1
```

(a)

```
divisor = 2
for i in range(0, 10, 2):
    print i/divisor
```

(b)

```
num = 10
while True:
    if num < 7:
        break
    print num
    num -= 1
```

(c)

```
count = 0
for letter in 'Snow!':
    print 'Letter #', count, 'is', letter
    count += 1
```

(d)

Q3: For the following, write the line(s) of code that will emit the given Output.

(15 Marks)

- ```
>>> a_list = [3, 5, 6, 12]
>>> YOUR CODE HERE
3
```
- ```
>>> a_list = [3, 5, 6, 12]
>>> YOUR CODE HERE
12
```
- ```
>>> a_list = [3, 5, 6, 12]
>>> YOUR CODE HERE
[5, 6, 12]
```



4. >>> a\_list = [3, 5, 6, 12]  
>>> YOUR CODE HERE  
3  
5  
6  
12
5. >>> a\_list = [3, 5, 6, 12]  
>>> YOUR CODE HERE  
[12, 6, 5, 3]
6. >>> a\_list = [3, 5, 6, 12]  
>>> YOUR CODE HERE  
[9, 15, 18, 36]
7. >>> a\_list = [3, 5, 6, 12]  
>>> YOUR CODE HERE  
[False, False, True, True]

**Q4: Explain the following codes and find the outputs:**

**(15 Marks)**

```
def greet(friend, money):
 if friend and (money >= 20):
 print "Good Friend"
 money = money - 10
 elif friend:
 print "Friend"
 else:
 print "Not Friend"
 money = money + 10
 return money
money = 20
money = greet(True, money)
print "Money:", money
print ""
money = greet(False, money)
print "Money:", money
print ""
money = greet(True, money)
print "Money:", money
print ""
```

```
questions = ['name', 'age', 'favorite color', 'city']
answers = ['ahmed', '20', 'blue']
for q, a in zip(questions, answers):
 print 'What is your {0}? It is {1}.'.format(q, a)
```

```
for x in range(1,10):
 print '{0:2d} {1:3d} {2:4d}'.format(x, 2*x, 3*x)
```

```
f = open('/workfile.txt', 'r+')
f.write('012345012345')
f.seek(5)
f.read(1)
f.seek(-3, 2)
f.read(1)
f.close()
```

*Good luck.*

*Dr. Abdelhameed Fawzy (6/2013)*

Mansoura University  
Faculty of Science  
Mathematics Department



4<sup>th</sup> level students (Mathematics / Statistics and computer Science programme)

Subject: Math 426 (Modelling and Simulations)

Final exam  
Second term  
May 2013

Date: 08 /06/2013

Time allowed: Two hours

Answer the following questions:

Total marks: 80

**Question one:**

- A) Write down and find the equilibria for each of the following difference equation models: the Beverton-Holt model, the Ricker model, and the Nicholson-Bailey model. **(10 marks)**
- B) Construct a mathematical model to describe the competition between two species. Explain your model, find the steady states and discuss their stability. **(10 marks)**

**Question two:**

- A) Write down a discrete-time model for measles with vaccination. **(5 marks)**
- B) Construct an SIS model for an infection spreading in a closed population with **varying** size. Explain your model, find the steady states and discuss their stability. **(15 marks)**

**Question three:**

- A) Write down both the von Bertalanffy and Gompertz models for the tumour growth. **(5 marks)**
- B) Prove that in a large **spherical** tumour there is a shell of proliferating cells, whose thickness depends on the excess nutrient concentration above a threshold  $(c_2 - c_1)$ , how fast the nutrient is consumed  $k$  and how fast it diffuses  $D$ , but not on the size of the tumour itself. **(15 marks)**

**Question four:**

**(20 marks)**

Construct an SIRS model for an infection spreading in a closed population with constant size and study the possibility to vaccinate the susceptible individuals with rate  $\psi$ . Find the critical age above which susceptible individuals should be vaccinated to protect the population from the infection.

Best regards,  
The examiner  
Dr. Muntaser Safan



**Answer QUESTIONS [1,2]**

**Question 1:[12 Marks]**

A- Use the Master method to solve the following recurrences

- i.  $T(3n)=T(2n/9)+\Theta(n^3)$       ii.  $T(n)=7T(n/2)+\Theta(n^2)$   
iii.  $T(n)=1.5T(2n/3)+\Theta(2^{\lg(n)})$

B- Use the substitution method to verify that the running time for the recurrence relation  $T(n)=9T(n/3)+n^2$  is of order  $O(n^2 \lg(n))$ .

**Question 2:[12 Marks]**

A- Write both the tilde approximations and the order of growth for the following:

- I.  $3N^2 - 150 \lg(N^3) - 12N$       II.  $(7N^3 - 3/N)(2^N - 12)$       III.  $7^N + \lg(N)$   
IV.  $2 \lg N^2 + 4 \lg^2 N$

B- Write the Merge sort algorithm and find its running time; then illustrate the behaviour of Merge Sort on the following array of integers {234, 344, 29, 438, 429, 37, 345, 23}.

**Answer ONLY TWO from QUESTIONS [3,4,5]**

**Question 3:[18 Marks]**

A- Use the recursion tree method to get the solution of the following recurrence.

$$T(3n)=T(5n/2)+T(3n/2)+O(n^2)$$

B- Use the recursion tree to find a good guess for the recurrence  $T(n)=7T(n/2)+n^2$ ; then verify your guess using the substitution method.

C- Compute the running time complexity of the following algorithm

```
function Sum (array A, int n)
{
 Sum=0
 for (i=0; i<n; i++)
 Sum=Sum+A[i]
 return Sum
}
```

**Question 4:[18 Marks]**

A- Write the Insertion sort algorithm and find both its best and worst running times; then illustrate the behaviour of Insertion Sort on the following array of integers {2, 44, 49, 8, 9, 57, 5, 3}.

B- Compute the running time complexity of the following algorithms:

```
function LSUM (N)
{
 int sum=0;
 for (int i=1; i < N; i++)
 for (int j=N; j > 0; j=j/2)
 sum+=j;
 return Sum
}
```



C- Analyze the following algorithm to find an asymptotic upper bound of its execution  
function multiply(x,y)

Input: Positive integers  $x$  and  $y$ , in binary

Output: The product of  $x,y$

$n = \max(\text{size of } x, \text{size of } y)$

if( $n=1$ ) return  $x,y$

$x_L =$  leftmost  $n/2$  bytes of  $x$ ,  $x_R =$  rightmost  $n/2$  bytes of  $x$

$y_L =$  leftmost  $n/2$  bytes of  $y$ ,  $y_R =$  rightmost  $n/2$  bytes of  $y$

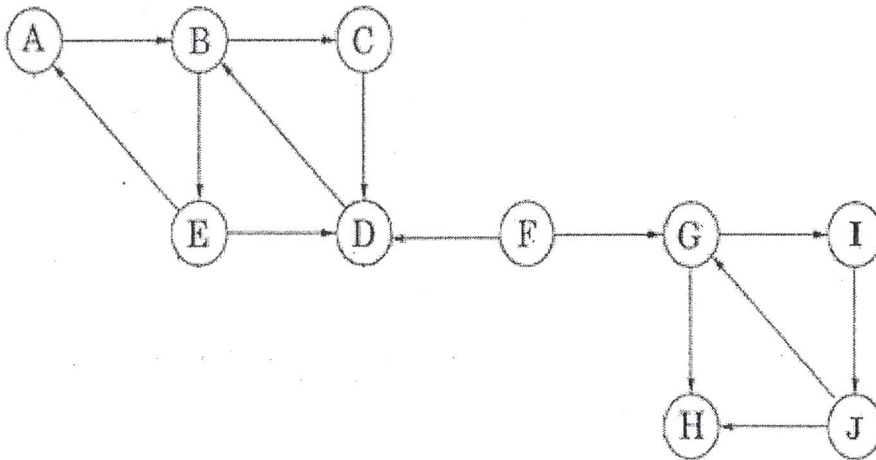
$P_1 = \text{multiply}(x_L, y_L)$

$P_2 = \text{multiply}(x_R, y_R)$

$P_3 = \text{multiply}(x_L + x_R, y_L + y_R)$

return  $P_1 \cdot 2^n + (P_3 - P_1 - P_2) \cdot 2^{n/2} + P_2$ .

Question 5:[18 Marks] Study the following graph algorithm, then answer the questions following it.



- Define the adjacency matrix for the graph.
- Write The Depth first Search Algorithm
- Perform Depth-First Search on the graph (starting from node A); *whenever there's a choice of vertices, pick the one that is alphabetically first*. Classify each edge as a tree edge, forward edge, back edge, or cross edge, and give the **first visit** and **final visit** order of each vertex., and state the complexity of its performance
- Draw the DFS search forest results from the DFS algorithm.
- Write an algorithm that identifies the strongly connected components within the graph.
- Write down the strongly connected components within the graph.

.....  
*End of Questions*



|                                                               |                                                                                                                                                                   |                                                                                             |
|---------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------|
| <p>دور مايو 2013<br/>الزمن: ساعتين<br/>التاريخ: 2013/6/13</p> |  <p>كلية العلوم - قسم الرياضيات<br/>برنامج: الرياضيات - الاحصاء وعلوم الحاسب</p> | <p>الفرقة: الثالثة<br/>المادة: دوال خاصة<br/>كود المادة: 324<br/>الدرجة الكلية: 80 درجة</p> |
|---------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------|

اجب عن الاسئلة الاتية

السؤال الاول: (20 درجة)  
احسب التكاملات الاتية

(6 درجات)  $\int_0^{\infty} e^{-t^2} dt$  - (ا)

(6 درجات)  $\int_0^{\infty} \frac{1}{1+y^4} dy$  - (ب)

(8 درجات)  $\Gamma(2x) = \frac{2^{2x-1} \Gamma(x) \Gamma(x + \frac{1}{2})}{\sqrt{\pi}}$  ان اثبت ان (ج)

السؤال الثاني: (20 درجة)

(7 درجات) (ا) - اثبت ان  ${}_2F_1(\frac{1}{2}, 1, \frac{3}{2}, z^2) = \frac{1}{2z} \log \frac{1+z}{1-z}$

(5 درجات) (ب) - اوجد  $P_3(x)$  حيث  $P_0(x) = 1$  و  $P_1(x) = x$   $P_n(x)$  هي كثيرة حدود ليجندر

(ج) - اوجد مفكوك الدالة  $f(x) = \begin{cases} 1 & 0 \leq x < 1 \\ 0 & -1 \leq x < 0 \end{cases}$

(8 درجات) في الصورة  $\sum_{m=0}^{\infty} A_m P_m(x)$

السؤال الثالث: (20 درجة)

(7 درجات) (ا) - اثبت انة عندما  $n$  عدد صحيح فان  $J_{-n}(x) = (-1)^n J_n(x)$

(6 درجات) (ب) - اوجد قيمة  $\int J_1(\sqrt[3]{x}) dx$

(7 درجات) (ج) - اثبت ان  $4J_0''' + 3J_0'' + 3J_3 = 0$

السؤال الرابع: (20 درجة)

(6 درجات) 1. اذكر صورة رديجيوس لكثيرات حدود لاجير  $L_n(x)$  ومن ثم اوجد  $L_2(x)$

(4 درجات) 2. اذكر خواص التعامد لكثيرات حدود لاجير

(10 درجات) 3. اثبت ان  $L_n(x) = n! {}_1F_1(-n, 1, x)$

مع اطيب التمنيات بالتوفيق والنجاح  
د. عبد المنعم لاشين

امتحان دور مايو ٢٠١٣ م  
برنامج : احصاء و علوم الحاسب  
المستوى : الرابع  
اسم المقرر : سلاسل زمنية و تنبؤ  
كود المادة : ر ٤٣٥



جامعة المنصورة - كلية العلوم  
قسم الرياضيات  
التاريخ : ٢٠١٣ / ٦ / ١٥ م  
الدرجة الكلية : ٨٠ درجة  
الزمن : ساعتان

**Answer the following questions:**

[1] a- Consider the MA(q) process given by  $X_t = Z_t + \theta_1 Z_{t-1} + \dots + \theta_q Z_{t-q}$ , where

$\{Z_t\} \sim WN(0, \sigma^2)$ . i) When  $q = 1$ , and  $|\theta_1| < 1$ , show that process can be

represented as  $Z_t = \sum_{j=0}^{\infty} (-\theta_1)^j X_{t-j}$  (10 marks)

ii) Show that  $|\rho(1)| \leq \cos\left(\frac{\pi}{q+2}\right)$  when  $q = 1$  and  $q = 2$  (10 marks)

b- Consider the time series model  $X_t = m_t + Y_t$ , where  $m_t$  is a polynomial trend of degree three and  $Y_t$  denotes the random noise component.

Find a seven point filter  $\{a_j\}_{j=-3, \dots, 3}$  (10 marks)

[2] Consider the process  $X_t - 0.4 X_{t-1} - 0.45 X_{t-2} = Z_t + Z_{t-1} + 0.25 Z_{t-2}$

i) Classify it as an ARMA (p, q) process (4 marks)

ii) Determine whether it is causal and / or invertible (6 marks)

iii) Find the coefficients  $\psi_j$  in its linear representation  $X_t = \sum_{j=0}^{\infty} \psi_j Z_{t-j}$  (10 marks)

iv) Use the linear process representation to drive a formula for its autocorrelation function  $\rho(h)$  and evaluate it for  $h = 1$  (10 marks)

[3]a- Prove that the Process  $X_t = X_{t-1} + Z_t$  where  $\{Z_t\} \sim WN(0, \sigma^2)$ , is not stationary process. (10 marks)

b- Define the following: i) The ARMA (p, q) process ii) White noise process

iii) Weak stationary time series iv) The operators  $\nabla$  and  $\nabla_d$

v) Non- negative definite function (10 marks)

Best wishes Dr. Faten Shiha