



Faculty of Science
Mathematics Department

بسم الله الرحمن الرحيم

3th Level Exam
Mathematics
Statistics & Computer Science

Integral Equations

May 2014

Time : 2 hours

Full mark 80

[1] a) Reduce the initial value problem

$$y'' - y' \sin x + e^x y = x, \quad y(0) = 1, \quad y'(0) = -1,$$

to a Volterra - type integral equation (10 marks)

b) Find the resolvent kernel of the integral equation

$$\phi(x) = 1 + \cos(x) + \int_0^x \frac{1 + \cos(x)}{1 + \cos(t)} \phi(t) dt \quad \text{and then find its solution} \quad (10 \text{ marks})$$

[2] Solve the integro-differential equation

$$\phi''(x) + \phi(x) + \int_0^x \sinh(x-t) \phi(t) dt + \int_0^x \cosh(x-t) \phi'(t) dt = \cosh x$$

$$, \phi(0) = -1, \phi'(0) = 1 \quad (20 \text{ marks})$$

[3] a) Using Fredholm determinant, find the resolvent kernel of the integral equation

$$\phi(x) = e^x - \int_0^1 e^{(x-t)} \phi(t) dt, \quad \text{and then find its solution} \quad (10 \text{ marks})$$

b) Solve the following integral equation

$$\int_0^x (t^n x^{n+1} - t^{n+1} x^n) \phi(t) dt = x^{2n}, \quad n = 2, 3, \dots \quad (10 \text{ marks})$$

[4] Find the characteristic values and Eigen functions of the integral equations

$$\phi(x) - \lambda \int_0^\pi (\cos^2 x \cos 2t + \cos 3x \cos^3 t) \phi(t) dt = 0 \quad (20 \text{ marks})$$

Best Wishes

Dr. Mahmoud Abdelaziz



Answer the following questions

Question 1:

(a) Show that A , the least squares estimator of the intercept α in the equation $\mu_{Y|x} = \alpha + \beta x$, is normally distributed with mean α and variance $\sigma_A^2 = \frac{\sigma^2}{n} \left(\frac{\sum x^2}{s_{xx}} \right)$. [9 points]

(b) Consider the following experiment data

x	10.0	10.5	11.0	11.5	12.0	12.5	13.0	13.5	14.0	14.5
y	16.7	19.5	16.5	17.6	16.2	18.8	19.6	20.4	21.3	17.6

- (i) Estimate the linear regression line. [6 points]
 (ii) Use the analysis of variance approach to test that $\beta = 0$ against the alternative hypothesis $\beta \neq 0$. At the 0.05 level of significance. [6 points]
 (iii) Construct a 95% confidence interval for y_0 when $x_0 = 10.8$. [6 points]

Hint: $\sum x = 122.5$, $\sum y = 184.2$, $\sum xy = 2268.65$, $\sum x^2 = 1521.25$, $\sum y^2 = 3420.8$, $f_{0.05}(1,8) = 5.32$, $t_{0.025}(8) = 2.306$

Question 2:

(a) For the following data

x	4.3	4.5	5.9	5.6	6.1	5.2	3.8	2.1	7.5
y	12.6	12.1	11.6	11.8	11.4	11.8	13.2	14.1	10.8

- (i) Compute and interpret the correlation coefficient between the variables. [7 points]
 (ii) Test the hypothesis that $\rho = 0$ against the alternative that $\rho \neq 0$. [7 points]

Hint: $\sum x = 45$, $\sum y = 109.4$, $\sum xy = 534.82$, $\sum x^2 = 244.26$, $\sum y^2 = 1337.86$, $Z_{0.05} = 1.96$

(b) Given the data

x	0	1	2	3	4	5	6	7
y	8.1	6.3	1.2	3.6	3.8	1.9	4.7	6.1

Fit a regression curve of the form $\mu_{Y|x} = \beta_0 + \beta_1 x + \beta_2 x^2$ and then estimate $\mu_{Y|2}$. [12 points]

Hint: $\sum x = 28$, $\sum x^2 = 140$, $\sum x^3 = 784$, $\sum x^4 = 4676$, $\sum y = 35.7$, $\sum xy = 115.1$, $\sum x^2 y = 619.9$

Question 3: Four laboratories are being used to perform chemical analysis. Samples of the same material are sent to the laboratories for analysis as part of the study to determine whether or not they give, on the average, the same results. The analytical results for the four laboratories are as follows:

	Laboratory				
	1	2	3	4	
	53.7	60.7	52.9	56.7	
	56.4	62.5	53.1	56.3	
	65.9	61.1	54.3	56.9	
	54.1	57.2	52.2	57.4	
	53.2	58.3	55.1	58.3	
$\sum y_i$	283.3	299.8	267.6	285.6	1136.3
$\sum y_i^2$	16164.51	17994.68	14327.36	16315.84	64802.39
\bar{y}_i	56.66	59.96	53.52	57.12	56.815
s_i^2	28.183	4.668	1.352	0.592	2.1747

- (a) Use Bartlett's test to show that the within-laboratory variances are not significantly different at the $\alpha = 0.05$ level of significance. [12 points]
 (b) Perform the analysis of variance and give conclusions concerning the laboratories. [14 points]

Hint: $b_4(0.05,5) = 0.585$, $F_{0.05}(3, 16) = 3.24$.



Answer the Following questions

1. Matlab as a calculator

[30 points]

Matlab can be used as a calculator, show the steps of evaluating

i. $1.11 + \frac{\sin(x + \Delta t) + 2\sin(x) + \sin(x - \Delta t)}{2\Delta t}$ at $x=1.5, \Delta t = 1$

ii. $z = 1 + 3(x - x_0) + (x - x_0)^3$ at $x_0=2, x = \begin{bmatrix} 1 \\ 1.5 \\ 2 \\ 3 \end{bmatrix}$

iii. $x = 1.3$; `ceil(x)`; `floor(x)`; `round(x)`;
Mention a case when to use each function.

2. Modeling

[30 points]

a) Write code to build a RobotCar class with:

[10 points]

position, velocity, and lanes properties,
the robotcar constructor, and

accelerate function that takes rate and time

Then construct an instance of the robot car class and accelerate it with rate = 5 and time =2

b) Draw the Simulink model for the following mathematical system

[10 points]

$$\frac{d}{dt} y(t) + 7y(t) = x(t)$$

$$x(t) = u(t), \text{ initial condition } y(0) = 1$$

c) Relation between Wavelength and Intensity

[10 points]

$$P_{\lambda} = \frac{2\pi hc^2}{\lambda^5 (e^{hc/\lambda kT} - 1)}$$

where P_{λ} is power per m^2 area per m wavelength, h is Planck's constant (6.626×10^{-34} J.s), c is the speed of Light (3×10^8 m/s), k is Boltzmann Constant (1.38×10^{-23} J/K), T is temperature (K), λ is the wavelength (m) and it has the range [10^{-9} , 3000×10^{-9}]

Write the code and plot the curve for: $T_1 = 4500, T_2 = 6000, T_3 = 7500$ K;

الفرقة: الثالثة

الشعبة: إحصاء وحاسب ورياضيات

المادة: دوال خاصة

324 ✓



كلية العلوم
قسم الرياضيات

دور مايو ٢٠١٤

الزمن: ساعتان

التاريخ: ١١ / ٦ / ٢٠١٤

الدرجة الكلية ٨٠ درجة

أجب عن الاسئلة الآتية:

(٢٠ درجة)

السؤال الأول:

$$\Gamma(2x) = \frac{2^{2x-1}}{\sqrt{\pi}} \Gamma(x) \Gamma(x + \frac{1}{2})$$

(i) أثبت أن

$$n! = \sqrt{2\pi} n^{n+\frac{1}{2}} e^{-n}$$

(ii) أثبت أن لقيم n الكبيرة

$$\int_0^2 (4-x^2)^{\frac{3}{2}} dx$$

(iii) احسب

(٢٠ درجة)

السؤال الثاني:

$$\frac{d}{dx} [x J_n(x) J_{n+1}(x)] = x [J_n^2(x) - J_{n+1}^2(x)]$$

(i) إثبت أن

$$x y'' - y' - x^2 J_0'(x) = 0 \quad \text{هي حل للمعادلة التفاضلية} \quad y = x J_1(x)$$

(ii) إثبت أن

(٢٠ درجة)

السؤال الثالث:

$$p_1(x) = x \quad \text{و} \quad p_0(x) = 1 \quad \text{إذا علم أن} \quad p_2(x) \quad \text{إيجاد العلاقات التكرارية لإيجاد}$$

$$\int_0^{\infty} e^{-x} (L_n(x))^2 dx$$

(ii) احسب

(٢٠ درجة)

السؤال الرابع:

$${}_2F_1\left(\frac{1}{2}, \frac{3}{2}; x^2\right) = \frac{1}{2x} \ln \frac{1+x}{1-x}$$

(i) إثبت أن

$$H_n''(x) = 4n(n-1)H_{n-2}$$

(ii) إثبت أن

تمنيتي بالنجاح والتوفيق

د/ مجدى يوسف برسوم

إن شاء الله



Answer the following questions: (Total grades: 60 G)

1. Consider a state space where the start state is number 1 and the successor function for state n returns two states, numbers $2n$ and $2n+1$. (15 G)
 - a. Draw the portion of the state space for state 1 to 15.
 - b. Suppose the goal state is 11. List the order in which nodes will be visited for breadth-first search, depth-limited search with limit 3, and iterative deepening search.
 - c. Would bidirectional search be appropriate for this problem? If so, describe in detail how it would work.
 - d. What is the branching factor in each direction of the bidirectional search?
 - e. Does the answer to (c) suggest a reformulation of the problem that would allow you to solve the problem of getting from state 1 to a given goal state with almost no search?
2. Describe briefly the basic types of the rational agents and draw their schematic diagrams? (10 G)
3. True or false - correct the wrong sentences (5 G):
 - a. In artificial intelligence, a human-centered approach involves a combination of mathematics and engineering to solve the intelligent problems.
 - b. For rational systems, computational limitations make perfect rationality unachievable. Then, we need to design the best program for the given machine resources.
 - c. Depth-first search always expands at least as many nodes as A* search with an admissible heuristic.
 - d. $h(n) = 0$ is an admissible heuristic for the 8-puzzle.
4. What is the type of problems that can be solved by using Artificial Neural Networks (ANNs)? List the main strengths, advantages, and disadvantages of ANNs? (8 G)
5. Answer briefly the following questions:
 - a) What is the difference between local search and uninformed/informed searches? (3 G)
 - b) What is the difference between human-centered and rational AI approaches? (4 G)
 - c) What is A* search? Is it complete and optimal? (4 G)
 - d) What is the Hill-climbing Search? When can it stuck? (4 G)
 - e) What is the difference between Conformant, Contingency, and Exploration Problems? (4 G)
 - f) What is the difference between Iterative Deepening Search and Iterative lengthening Search? Is iterative deepening search wasteful? (3 G)

With my best wishes
Dr. Mohammed Elmogy

Mansoura University
Faculty of Science
 Mathematics Department
 Statistics and Computer Science



Operating System
 Concepts (٣٤٥)
 Sun: 1st Jun, 2014
 Dr. Bahaa Shabana

Answer Five (5) questions from the following.

1. Write short note for each of the following:

- | | |
|-----------------|------------------|
| 1.1. MMU. | 1.4. Dispatcher. |
| 1.2. PCB. | 1.5. Swapping. |
| 1.3. Scheduler. | 1.6. Claim edge. |

2. Question Two

- 2.1. List and briefly define the four main elements of a computer.
- 2.2. What are the three main purposes of an operating systems?
- 2.3. What are the different states of the process? (Draw a diagram).

3. Question Three

- 3.1. What advantage is there in having different time-quantum sizes at different levels of a multilevel queuing system?
- 3.2. Name two differences between logical and physical addresses.
- 3.3. What elements are typically found in a page table entry? Briefly define each element.

4. Question Four

Consider the following set of processes, with the length of the CPU burst given in milliseconds:

Process	Burst Time	Arrival Time	Priority
P_1	6	0	5
P_2	4	3	1
P_3	2	2	3
P_4	1	4	4

- 4.1. Draw three Gantt charts that illustrate the execution of these processes using:
 - 4.1.1. FCFS algorithm.
 - 4.1.2. SJF preemptive algorithm.
 - 4.1.3. RR algorithm, with quantum = 3.
 - 4.1.4. Non-preemptive priority (a smaller priority number implies a higher priority).
- 4.2. Calculate the average waiting time for each algorithm.
- 4.3. Which algorithm is the best?

(Please see next page)

5. Question Five

Consider the following page reference string:

1, 2, 3, 4, 2, 1, 5, 6, 2, 1, 2, 3, 7, 6, 1, 2, 3, 6.

- 5.1. Complete a figure showing *the 3 (Three) frame* allocation for:
 - 5.1.1. FIFO (first in first out) replacement.
 - 5.1.2. LRU (least recently used) replacement.
 - 5.1.3. Optimal replacement.
 - 5.2. How many pages fault for each replacement algorithm?
 - 5.3. Which algorithm is the best?
-

6. Question Six

- 6.1. What are the advantages and disadvantages for each of:
 - 6.1.1. Contiguous allocation.
 - 6.1.2. Linked allocation.
 - 6.1.3. Indexed allocation.
 - 6.2. Using a Resource-Allocation graph, give example of:
 - 6.2.1. Resource-Allocation graph with a deadlock.
 - 6.2.2. A cycle graph but no deadlock.
-

(End of Questions)

Good Luck

Dr. Bahaa Shabana