بحوث عمليات   الاحصاء وعلوم الاحصاء

٤٢١

المستوى الرابع رياضيات

Mansoura University *Faculty of Science* *Depart of Mathematics* *4th year* *Course: OR*	MANSOURA UNIVERSITY	*29-12- 2013* *Code :R421* *Time : 2 Hours* *Full Mark : 80*

NO. of Questions :4 — **Final Examination** — *NO. of Pages:2*

Answer all the following Questions

Question:1 (25 marks)

(a) If it is possible, solve the following mathematical models ***by using graphical method***

(i)
$$\begin{aligned} \text{Minimize} \quad & Z = x_1 + 4x_2 \\ \text{subject to} \quad & x_1 + 2x_2 \le 8, \\ & 2x_1 + x_2 \ge 4, \\ & x_1, x_2 \ge 0. \end{aligned}$$

(ii)
$$\begin{aligned} \text{Maximize} \quad & Z = 4x_1 + x_2 \\ \text{subject to} \quad & 3x_1 + x_2 \le 6, \\ & x_1 - x_2 \ge 2. \\ & x_1, x_2 \ge 0. \end{aligned}$$

(iii)
$$\begin{aligned} \text{Maximize} \quad & Z = x_1 + 5x_2 \\ \text{subject to} \quad & 3x_1 + 2x_2 \le 6, \\ & x_2 \ge 1, \\ & x_1 + x_2 \ge 1, \\ & x_1, x_2 \ge 0. \end{aligned}$$

(iv)
$$\begin{aligned} \text{Minimize} \quad & Z = x_1 + 5x_2 \\ \text{subject to} \quad & 2x_1 + x_2 \le 8, \\ & x_1 + x_2 \ge 4, \\ & x_1 \le 3, \\ & x_1, x_2 \ge 0. \end{aligned}$$

(b) Express the following linear programming problem in ***the standard form:***

$$\begin{aligned} \text{Minimize} \quad & Z = 3x_1 + x_2 + 15x_3 \\ \text{subject to} \quad & 2x_1 + x_2 - x_3 \le 4, \\ & 4x_1 - x_2 + 2x_3 \le -5, \\ & 3x_1 - x_2 + x_3 \ge 3, \\ & x_2, x_3 \ge 0. \end{aligned}$$

Question:2 (20 marks)

(a) Use ***The Simplex method*** to solve the following linear programming problem

$$\begin{aligned} \text{Maximize} \quad & Z = 3x_1 + 4x_2 \\ \text{subject to} \quad & x_1 + x_2 \le 3, \\ & 2x_1 + x_2 \le 4, \\ & x_1, x_2 \ge 0. \end{aligned}$$

(b) Use ***Two Phase method*** to solve

$$\text{Maximiz } Z = 5x_1 + 3x_2$$
$$\text{subject to } \quad x_1 + x_2 \le 1,$$
$$x_1 - x_2 \ge 6,$$
$$x_1, x_2 \ge 0.$$

Question:3 ***(20 marks)***

(a) ***Find the initial basic feasible solution to the following transportation problem by:***

(i) minimum cost method,

(ii) north-west corner rule,

(iii) Vogel's approximation method

To

		M_1	M_2	M_3	M_4	M_5	***supply***
	S_1	*17*	*4*	*8*	*7*	*2*	*60*
from	S_2	*13*	*5*	*16*	*12*	*0*	*30*
	S_3	*10*	*10*	*14*	*11*	*6*	*20*
	S_4	*3*	*9*	*19*	*8*	*18*	*40*
Demand		*40*	*25*	*35*	*30*	*20*	

Question:4 ***(15 marks)***

Solve the following Assignment problem:

	I	*II*	*III*	*IV*	*V*
1	*8*	*3*	*7*	*16*	*9*
2	*9*	*5*	*2*	*8*	*10*
3	*7*	*2*	*8*	*4*	*1*
4	*18*	*9*	*6*	*17*	*15*
5	*9*	*4*	*12*	*11*	*8*

GOOD LUCK

المستوى الرابع - رياضيات - نظرية المعادلات التفاضلية ، ٤١١

Faculty of science
Math-department

theory of differential equations
B.sc. Exam

January : 2014
Time : 2 H

Answer 4 questions out of the following :

1-a) Give the general solution and the path of the system $\begin{cases} \dfrac{dx}{dt} = 3x + 2y, \\ \dfrac{dy}{dt} = -5x - y \end{cases}$. (12marks)

b) Define the following : Lipschitz condition , Sturm – Liouville problem, fundamental matrix , the orthonormal functions and open connected domain .. (8marks)

2a) Prove that the function $f(x, y) = x \sin y + y \cos x$ satisfies Liptischitz condition w.r. to y in $D := \{(x, y) : |x| \le a, |x| \le b\}$. (5marks)

b) Prove that for the I.V .problem $\dfrac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$ their exist a unique solution $\varphi(t)$ on the domain $D := a < x < b$, $-\infty < y < \infty$. (10marks)

c) Give the solution of the I.V . problem $\dfrac{dy}{dx} = x^3 + y^3 + x + 1$, $y(0) = 0$. (5marks)

3 a) i-If the n vector functions $\varphi_1, \varphi_2, \ldots, \varphi_n$ are linearly dependent solutions of the H.L.V.D.E $\dfrac{dX}{dt} = AX$, $y(x_0) = y_0$ on $I = [a,b]$. Prove that the Wronskian $W(\varphi_1, \varphi_2, \ldots, \varphi_n)(t) = 0$

ii) Let $\varphi_1, \varphi_2, \ldots, \varphi_n$ be linearly dependent n solutions of the H.L.V.D.E $\dfrac{dX}{dt} = AX$.
if $W(\varphi_1, \varphi_2, \ldots, \varphi_n)(t_0) = 0$ at some point $t_0 \in I$. *Prove that* $\varphi_1, \varphi_2, \ldots, \varphi_n$ are linearly dependent solutions . (15marks)

b) Prove that the matrix $\begin{pmatrix} e^t & e^{2t} & e^{-3t} \\ e^t & 2e^{2t} & 7e^{-3t} \\ e^t & e^{2t} & 11e^{-3t} \end{pmatrix}$ is a fundamental matrix of the H.L.V.D.E.

$$\frac{dX}{dt} = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & -4 \\ 4 & 1 & -2 \end{pmatrix} X .$$ (5marks)

4-a) for the H.L.V.D.E $\dfrac{dX}{dt} = A(t)X$ and its fundamental matrix $\Phi(\tau)$, prove that the unique solution $\varphi(t)$ can be expressed in the forms :

i) $\varphi(t) = \Phi(\tau)\, C$

ii) $\varphi(\tau) = \Phi(\tau)\, \Phi^{-1}(\tau_0)\, x_0$. where $C = \begin{pmatrix} c_1 \\ \vdots \\ c_n \end{pmatrix}$, $\varphi(t_0) = x_0$. (14marks)

P.T.O

رياضيات / ر٤١٦ هندسة تفاضلية

دور: يناير : 2014 الزمن : ساعتان التاريخ : 1 /1/ 2014	كلية العلوم – قسم الرياضيات	الفرقة: المستوى الرابع المادة : هندسة تفاضلية كود المادة : ر416 البرنامج : رياضيات

Answer the following questions: *Full mark : 80*

1-a) Define the regular curve , the arc-length of a curve and find T , N , B and the arc length function based at t = 0 for the curve

$$\alpha(t) = (\cosh t \ , \sinh t \ , \ t)$$

b) Define the normal and the osculating planes for a space curve at a point and find their equations for the curve

$\alpha(t) = (a\cos t \ , a\sin t \ , bt)$ **at the point p = (a , 0 , 0).**

c) Prove that a regular curve is a plane curve iff its tortion $\tau \equiv 0$

2- a)Define T , N , τ **for a space curve** $\alpha(t)$ **and prove that**

$N = \dfrac{dT}{dt} \Big/ \left|\dfrac{dT}{dt}\right|$ **and** $\tau = (\dot{\alpha} \times \ddot{\alpha} \cdot \dddot{\alpha}) \Big/ |\dot{\alpha} \times \ddot{\alpha}|^2$.

b) Deduce the expression of the second fundamental form II of a surface and find II for the surface

$r(u, v) = (f(u)\cos v \ , \ f(u)\sin v \ , h(u))$.

3-a) For a space curve $\alpha(s)$ **prove that** $\dfrac{dB}{ds}$ **is parallel to N and find Frenet equations for a space curve** $\alpha(s)$.

b) Define the normal curvature K_n **and the Gaussian curvature** K **of a regular surface and find them for the cylinder**

$$r(u, v) = (a\cos v \ , \ a\sin v, u)$$

4- a) Evaluate Frenet apparatus for the curve

$\alpha(t) = (t \ , \frac{1}{2}t^2 \ , \frac{1}{6}t^3)$.

b) Define the tangent vector , the tangent plane $T_p(S)$ **of regular surface at some point p . Find the equation of the tangent plane of the surface**

$z = \frac{1}{2}(ax^2 + by^2)$ **at p (0 , 0 , 0).**

Best wishes
Dr / Awatif shahin

٤ رياضيات - جبر / ٤١٥

Jan. 2014 Time: 2 hours Lie algebra Math 415 Data: /1/2014	كلية العلوم قسم الرياضيات	Mans. Univ Faculty of Science Dept. Math.

Answer the following Questions:

[1] Let L be a Lie algebra. Define what we mean by V is L – module, where V is finite dimensional vector space

a) Give an example of a module structure of by L

b) Define what we mean by a L – module V is irreducible and also reducible.

[2] Let A be an algebra. A derivation D of A is an linear map such that $D(an) = aD(b) + D(a)b$ for all $a, b \in A$

a) Show that $[D, E] = DoE - EoD$ is a derivation where E is also a derivation.

b) Show also that $adx : L \to L$ is also derivation

[3] Define what we mean by L is a solvable Lie algebra.
Prove that every sub algebra and homomorphic image of L are solvable

[4]-a) Define what we mean by a map ϕ is a homomorphism from Lie algebra L_1 to a Lie algebra L_2

b) Define the mapping $ad : L \to g\ell(L)$ by $x \to ad\ell(x)$ such that $ad(x)(y) = [x, y]$ Prove that ad(x) is a homomorphism called adjoint homomorphish.

Good luck

٤١٣ / تحليل عددي ٥ - ع الاحصاء وعلوم الحاسب الرياضيات

Mansoura University, Faculty of Science, Mathematics Department
Numerical Analysis (2) Final Exam (Math. 413) - Term 1, January 2014
Fourth year students (Mathematics & Statistics and Computer Science) Time Allowed: 2 hours

مسموح باستخدام الآلة الحاسبة . Answer the following questions . All questions carry equal marks

---

Question no.1:

1-a) Establish the recurrence relation: $x_{n+1} = \frac{1}{2}(x_n + \frac{a}{x_n})$. Hence, or otherwise, compute $\sqrt{21}$ correct to 3 decimal places .

1-b) Compute $(3)_{-3}$, $(1)_{-5}$ and the Stirling matrices s_5 and S_5.

1-c) Find $\sum_{r=1}^{n} r(r-1)(r-2)(r-3)(r-4)$.

+

Question no. 2:

2-a) Find an approximate value of the root of the equation $f(x) = x^3 + x - 1 = 0$ by using

(i) The false position method formula twice,

(ii) The fixed point iterations.

(Hint: f(- 0.5) = - 0.375 and f(1) = 1).

2-b) Compute one root of $e^x - 3x = 0$ correct to two decimal places by using the bisection method.

Question no. 3:

3-a) Express $f(x) = x^4 - 5x^3 + 3x + 4$ in terms of factorial polynomials.

3-b) By using a suitable algorithm , find an approximated value for y at x=0.1 , given that $y(0) = 0.2$,

$$\left.\frac{dy}{dx}\right|_{x=0} = 0.5 \text{ and } \frac{d^2y}{dx^2} + 2x\frac{dy}{dx} - 4y = 0.$$

= = = = = **END OF EXAM** = = = = =

With kind regards

Examiner: Prof. Dr Moawwad El-Mikkawy

المستوى الرابع رياضيات - ميكانيكا الكم / ٤٢٣

Mansoura Univ.
Faculty of Science
Mathematics Dept.
Subject:Math.
Course Quantum Mechanics

4thYear: math.
Date Jan.2014
Time: 2 hours
Full marks: 80

Answer the following questions:

[1] a) Why classical mechanics is not suitable to study small particles (which are affected by emission or absorption of light)? Derive Schrodinger equation.

b) Derive Ehrenfest theorem. Explain its importance. [20 marks].

[2] Solve Schrodinger equation for the radial part of the hydrogen atom. [20 marks].

[3] a) Write short notice on quantum entanglement. Comment on Einstein objection to quantum mechanics.

b) Explain the perturbation method. Use it to explain Zeeman effect. [20 marks}.

[4] a) Derive uncertainty principle for unitary operators.

b) Prove that the Hamiltonian is a self adjoint operator. Why this is important?. [20 marks].

٤١٨/

جامعة المنصورة	المستوى الرابع (علوم)	الفصل الدراسي الاول 13-14
كلية العلوم	شعبة : الرياضيات وشعبة الاحصاء وعلوم الحاسب	التاريخ : 22/ 1 / 2014
قسم الرياضيات	المادة : نظرية الشبكات (ر418) (Lattice Theory "Math 418")	الزمن: ساعتان

Answer the following ~~five~~ questions:

1- Give two distinct equivalent definitions of meet-semi-lattices. (5 points)
And then give an example of each of: (each item 3 points)
1- A partially ordered set (poset) but not a semi-lattice with 4 elements.
2- A smallest non- modular lattice,
3- A modular lattice but not distributive with 6 elements.
4-Two distributive lattices having 7 elements.
5- An $\leq$ - homomorphism between two lattices but not $\wedge$- homomorphism.

2- a- Let $(L, \wedge)$ be a meet-semilattice as an algebra. (10 points)
Find a semilattice $(L, \leq)$ as a poset equivalent to $(L, \wedge)$.
Give an example of a $\wedge$-semilattice but not $\vee$-semilattice.

b- Give all $\vee$-semilattices with 4-element set. (5 points)
c- Give an example of a lattice, but neither modular nor distributive. (5 points)

3- For a commutative group $\boldsymbol{G} = (G ; \cdot)$.
a- Show that the set of all subgroups $S_N(G)$ of G containing the subgroup N forms a lattice. (10 points)

b- Find the lattice of all subgroups of the group $(Z_{12} ; +)$. (10 points).
Is the lattice of the subgroups of $(Z_{12} ; +)$. distributive? why?

4- a- Let a , x, y be any three elements in a lattice $\boldsymbol{L} = (L ; \vee , \wedge)$. (8 points)
Prove that:
$\boldsymbol{L}$ is distributive $\Leftrightarrow$ " $a \wedge x = a \wedge y$ & $a \vee x = a \vee y \Rightarrow x = y$ ".

b- Give two equivalent definitions of a $\vee$-ideal of a lattice $\boldsymbol{L} = (L ; \vee , \wedge)$ and prove the equivalence between them. (4 points)

c – Let θ be a congruence relation on a lattice $\boldsymbol{L} = (L ; \vee , \wedge)$. (8 points)
Show that each congruence class $[a]\theta$ is a convex sublattice of $\boldsymbol{L}$.
And show also that $[0]\theta$ is a $\vee$-ideal.

Examiner : Dr. Magdi H. Armanious **Full Mark: 80 points**