المستوى الثالث - رياضيات / ٣١١
احصاء وحاسب - نظرية القياس

Final Exam- Semester I - Year 2013/2014

SUBJECT: *Measure Theory* *(MATH 311)* Level-3	Faculty of Science Mathematics Department	DATE: 26 / 12 /2013 FULL MARK: *80* ALLOWED TIME: *2Hours*

Answer the following questions

Question-1 (20 marks)

1. Define the Algebra Ω of subsets of a set X. prove that, if Ω is an algebra and $A \in \Omega$, then $A^c \in \Omega$, and any ring Ω with this property is an algebra
2. Define the outer measure on an algebra Ω. And prove that If $\mu^*(A) = 0$, then $\mu^*(A \cup B) = \mu^*(B)$

Question-2 (20 marks)

1. Define the measurable set. And prove that if E is measurable set then the complement E^c is also measurable
2. Prove that, the cantor set is measurable and find its measure

Question-3 (20 marks)

1. Define the measurable function. And prove that every continuous function is measurable
2. Prove that if f_1 and f_2 are measurable on [a,b] then so are $\{f_1\}^2$, $f_1.f_2$ and $Min\{f_1, f_2\}$

Question-4 (20 marks)

1- Show that the function

$$f(x) = \begin{cases} 1, & x \text{ is an irrational number in} [-4,4] \\ -2, & x \text{ is a rational number in} [-4,4] \end{cases}$$

a) Is not Riemann integrable in [-4,4]
b) Is Lebesgue integrable in [-4,4] and find the value of Lebesgue integral of f(x) in [-4,4]

2- Prove that if f(x)=c then $\int_E f(x)dx = c\mu(E)$

المستوى الثالث - رياضيات - ر ٣١٣ تحليل عددي (١)
الاصدار بلون الاب

Level: 3 Program: Mathematics + Statistics & Computer Science Numerical Analysis (1) (313ر)	Faculty of Science Mathematics Department	1^{st} Semester Time: 2 hour Date: 30/12/2013

Answer the following Questions:

Question (1)

(1) Derive Newton-Raphson method, and use it to find an approximation to $\sqrt[3]{25}$ correct to within 10^{-3}. (10 Marks)

(2) Define Lipschitz condition. Show that the following initial-value problem has a unique solution. Use Runge-Kutta method of order four to approximate it.

$$y' = y - t^2 + 1, \quad 0 \le t \le 1, \quad y(0) = 0.5, \text{ with } h = 0.2$$ (10 Marks)

Question (2)

(1) State and prove under what condition(s) the function g(x) has a unique fixed point in $[a, b]$. (10 Marks)

(2) Show that the sequence defined by

$$x_n = \frac{1}{2} x_{n-1} + \frac{1}{x_{n-1}}, \quad \text{for } n \ge 1,$$ (10 Marks)

converges to $\sqrt{2}$ whenever $x_0 > \sqrt{2}$

Question (3)

(1) State and derive Newton Forward-Difference Formula. (5 Marks)

(2) By using the following data (15 Marks)

x	1	1.05	1.1	1.15	1.2
f(x)	0.1924	0.2414	0.2933	0.3492	0.3943

Find an approximate value for

(i) $f(1.01), f(1.09), f(1.18)$, (ii) $f'(1.1)$, using 3 and 5-point formula

(iii) $\int_1^{1.2} f(x)\,dx, \quad n = 3, 4.$

Best Wishes; *Dr. Tamer Mohamed El-Azab*

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الفصل الدراسى الاول الزمن : ساعتان التاريخ:٢/ ١/ ٢٠١٤	كلية العلوم قسم الرياضيات	المستوى: الثالث البرنامج : الرياضيات اسم المقرر: ر٣١٢تحليل مركب

Answer the following questions:

[1]-a) Define : Simple open contour, analytic function at z_0, simply connected domain. (4 Marks)

b) If $f(z)=u(x,y)+iv(x,y), z_0=x_0+iy_0, w_0=u_0+iv_0$. Then prove that $\lim\limits_{z\to z_0} f(z)=w_0$ if ana only if $\lim\limits_{(x,y)\to(x_0,y_0)} u(x,y)=u_0$ and $\lim\limits_{(x,y)\to(x_0,y_0)} v(x,y)=v_0$. (10 Marks)

c) Prove that $\cos^4\theta=\frac{1}{8}\cos 4\theta+\frac{1}{2}\cos 2\theta+\frac{3}{8}$. (6 Marks)

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[2]-a) State and prove the necessary C. R. E's for w = f(z) to be differentiable at z_0. (12 Marks)

b) Prove that $f(z)=-e^{2x}\sin 2y+ie^{2x}\cos 2y$ is an entire function. (8 Marks)

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[3]-a) Show that u(x,y) = cos x cosh y is harmonic function and find its harmonic conjugate v(x, y) so that $f=u+iv$ is analytic. (7 Marks)

b) State without proof Laurent's theorem. (6 Marks)

c) If w = f(z) is analytic and $f'(z)$ is continuous. Then prove that $\int_C f(z)\,dz=0$. (7 Marks)

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[4]-a) Let f(z) be analytic on and in the interior of a simple closed contour C, $z_0\in$ Int (C). Then prove that

$$f(z_0)=\frac{1}{2\pi i}\int_C \frac{f(z)}{z-z_0}\,dz$$ (7 Marks)

b) Find $\int_{C_+} \frac{\cos z}{(z-1)^3(z-5)^2}dz,\ C_+:|z-4|=2$. (6 Marks)

c) Define pole of order m. Expand the function $f(z)=\frac{e^{\frac{1}{z-1}}}{z-1}$ about z=1 and dissus the singnlarity of this function. (7 Marks)

الممتحن أ.د محمد كمال عبد السلام عوف

الرياضيات / ٣٢١ نظرية الاحتمالات (١)

الاحصاء والحاسب

Mansoura University *Faculty of Science* *Math. Dept.*		***Exam :*** 9^{th} ***Jan.*** **2014** **Time : 2 hours** **3rd year (Stat. & Comp. Sci.)**

Subject : Probability Theory

Answer the following questions: ***(Total Mark 80)***

(1) a- If X_1 and X_2 are independent exponential random variables with respective parameters λ_1 and λ_2.

Find (i) the distribution of $Z = X_1 / X_2$.

(ii) $P(X_1 < X_2)$. (12 M.)

b- If X is a random variable with finit mean μ and variance σ^2, then for any value $k > 0$, prove that $P\left\{|x-\mu| \geq k\right\} \leq \dfrac{\sigma^2}{k^2}$. (8 M.)

(2) a- If X_1 and X_2 are independent random variables having Poisson distribution with parameters λ_1 and λ_2. Find the probability distribution of the random variable $Y = X_1 + X_2$. (10 M.)

b- Suppose the joint density of X and Y is given by

$$f(x,y) = \frac{e^{-x/y} e^{-y}}{y} \qquad 0 < x < \infty, 0 < y < \infty$$

(10 M.) $P(X \succ 0|Y = y)$.. ii) $E(X|Y = y)$ Find i)

(3) a- For any two random variables X and Y , show that

$Var(X+Y) = Var(X) + Var(Y) + 2Cov(X,Y)$. (5 M.)

b- If X is a binomial random variable with parameters $n \& p$.

Find i - $\phi_X(t)$, the characteristic function.

ii- Var (X). iii- α_3, the skewness. (15 M.)

(4) a- If X is a random variable having the moment (GF) $M_X(t)$ and μ'_n is the n–th moment, prove that $\mu'_n = \dfrac{d^n}{dt^n} M_X(t)|_{t=0}$ (7 M.)

b- For any random variable X, prove that :

$$E(X) = \int_0^\infty P\{X > x\}\, dx - \int_0^\infty P\{X < -x\}\, dx \,. \qquad \text{(7 M.)}$$

c- A proup of n men throw their hats into the center of a room. The hats are mixed up and each man randomly selects one. Find the expected number of men that select their own hats (called matches). (6 M.)

Best wishes.

Prof. Beih El-Desouky

جامعة المنصورة	المستوى الثالث	الفصل الأول
كلية العلوم	شعبة رياضيات	يناير 2014
قسم الرياضيات	المادة: ميكانيكا تحليلية ر 326	الزمن: ساعتان

أجب عن الأسئلة التالية:

1- أ) اذكر شروط تطبيق ميكانيكا لاجرانج على المنظومة الميكانيكية.

ب) اذكر شرط وجود صياغة لاجرانجية لمعادلات حركة منظومة معطى لها دالة هاملتون $H(q_1,...,q_n,p_1,...,p_n)$.

ج) - خرزة تتحرك على سلك دائرى أملس مستواه رأسى بينما يدور السلك بسرعة زاوية ثابتة ω حول قطره الرأسى، أوجد دالة لاجرانج وحل معادلة الحركة مستعينا بالتكامل الأول للحركة.

2- أ) عرف منظومة ليوفيل وبين كيفية حل مسألة حركتها بفصل المتغيرات .

ب) جسيم كتلته الوحدة يتحرك فى المستوى تحت تأثير قوى جهدها $V=-(\frac{\mu_1}{r_1}+\frac{\mu_2}{r_2})$ (الجذب النيوتونى لمركزين ساكنين المسافة بينهما $2a$). بين أن المنظومة تأخذ شكل منظومة ليوفيل فى الإحداثيات الناقصية وبين كيفية حل مسألة الحركة بفصل المتغيرات .

3- منظومة لها دالة لاجرانج $L=\frac{1}{2}\dot{v}^2+v\dot{u}\dot{v}-V(v)$

أ) أوجد التكاملات الأولى لحركة المنظومة.

ب) عبر عن وضع المنظومة (الإحداثيين u,v) بدلالة الزمن.

ج) بين مع التعليل ما إذا كانت هذه المنظومة تكافئ منظومة راوث أو هاملتون وأوجد دالة راوث أو هاملتون المناظرة.

أستاذ المادة: أ. د./ حمد حلمى يحيى .

المستوى الثالث رياضيات - توبولوجي (١) ، ٣١٦

Mansoura University Faculty of Science Department of Mathematics Final term exam		Topology Third Level (Mathematics) Time: 2 hours 23 / 1 / 2014

<u>Answer the following questions:</u>

1- (i) Prove that A is a closed set iff $A' \subseteq A$.

(ii) Show that the denseness is a topological property.

2- (i) Prove that the topological space (X,τ) is T_1 – space iff $\{x\}$ is a closed set for every $x \in X$.

(ii) Let (X,C) be the co-finite topological space and $p \in X$. Show that $N_p \subseteq C$, where N_p is the neighborhood system of the point p.

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(3)- (i) Prove that the mapping $f:(X,\tau) \to (Y,\delta)$ is continuous iff $\overline{f^{-1}(B)} \subseteq f^{-1}(\overline{B}), \forall B \subseteq Y$.

(ii) Let (X,τ) be a T_2- space and $Y \subseteq X$. Prove that (Y,τ_Y) is T_2- space, also.

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(4) Let N be the set of natural numbers and τ be the family $\{\phi, E_n : n \in N\}$, where $E_n = \{n, n+1, n+2, n+3, \ldots\}, n \in N$.

Show that the family τ is a topology on N. Also, find: the closure, the interior, the boundary, and the derived of the sets $A = \{2,4,6,\ldots\}$ and $B = \{5,9,13\}$.