دور مايو 2015		الفرقة: الرابعة
الزمن: ســاعتــان		الشعبة: رياضيات+إحصاء وعلوم الحاسب
التاريخ: 2015/5/16	كليـــة العلوم – قسم الرياضيات	المادة: تحليل دالي

Answer the following questions

First : Objective questions : (20 marks)

Among the following statements mark the true and false ones with (√) and (×) respectively. Justify your answer for ONLY TWO of them :

(a) In a metric space X, for any set $A \sqsubseteq X$ we always have $\overline{\overline{A}} = A$. ....( )

(b) A complete metric space is a Banach space. ....( )

(c) Any subspace of a Banach space is also a Banach space. ....( )

(d) The sequence $\left(1/\sqrt{n} \right)$ belongs to the space ℓ^3. ....( )

(e) If $A = \{x_1, x_2, x_3, x_4, x_5, x_6\} \subseteq \ell^\infty$, it follows that the linear hull H(A) is separable. ....( )

(f) For any normed space E over K, the space L(ℓ^∞, E) is a Banach space. ....( )

(g)A normed space E that is linearly isometric to a Banach space F, is itself Banach space. ....( )

(h) For any normed space E over K, the dual space E^* is a Banach space. ....( )

(i) Every linear transformation $S: K^5 \to \ell^\infty$ is continuous on R^5. ....( )

(j) If E is a normed space over K, then any two norms defined on E are equivalent. ....( )

Second : Subjective questions (26 marks each)

[2] a. Define: a metric space - the space ℓ^p. (4 marks)

Let $p > 1$, $q = p/(p-1)$. Prove that if $(\alpha_n) \in \ell^p, (\beta_n) \in \ell^q$, then the series $\sum_n \alpha_n \beta_n$ is absolutely convergent ; and

$$\sum_{n=1}^{\infty} |\alpha_n \beta_n| \le \left(\sum_{n=1}^{\infty} |\alpha_n|^p \right)^{1/p} \cdot \left(\sum_{n=1}^{\infty} |\beta_n|^q \right)^{1/q}.$$ (10 marks)

b. Show that a convergent sequence in a metric space has a unique limit. (6 marks)

[3]] a. Define : a separable space. (2 marks)

Show that the space ℓ^p is separable ($p \ge 1$). (8 marks)

b.Let E,F be normed spaces over K , $E \neq \{0\}$, and let $T: E \to F$ be a linear mapping of E onto F. Prove that that T is 1-1 and T^{-1} is bounded <u>if and only if</u> there exists a constant $m > 0$ such that $\|Tx\| \ge m$,for all $x \in E$ with $\|x\| = 1$. (10 marks)

[4] a. Consider the mapping $T: R^2 \to R^3$, $T(a,b) = (2a+b, a-2b, b)$ for all $(a,b) \in R^2$

Show that T is a bounded linear transformation on R^2 and then find $\|T\|$. (9 marks)

b. Let E,F be normed spaces over K and suppose that E is finite-dimensional. Prove that :

(i) E is a Banach space , and (ii) every linear transformation $T: E \to F$ is continuous on E.

(3+8 marks)

Best wishes..Ever

Samir Kassem

Mathematics department **Date : 19-5-2015** **Time : 2 hours** **Full Mark : 80**		**4th. Final exam.** **Mathematics group** **Partial differential** **Equations 429** ر

<u>Answer the following questions:</u>

[1] a) Find D' Alembert solution of the wave equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ subject to the B. C's $u(x,0) = f(x)$ and $\left(\frac{\partial u}{\partial t}\right)_{t=0} = g(x)$ and show that the solution is unique and stable. (10 Marks)

b) Show that the solution $u(x,y)$ of Laplace's equation $\nabla^2 u = 0$ in the region $0 < y < a$, $x > 0$ satisfying $u(x,0) = f(x)$ and $u(x,a) = 0$ where $f(x)$ is a given function and a is a constant is

$$u(x,y) = \frac{1}{\pi}\int_0^\infty \left[\int_{-\infty}^{\infty} \frac{\sinh\lambda(a-y)}{\sinh\lambda a} f(\xi)\cos\lambda(\xi - x)d\xi\right]d\lambda$$ (15 Marks)

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[2] Show that the solution of the equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t}$ which satisfies the conditions

$\left(\frac{\partial u}{\partial x}\right)_{x=0} = \left(\frac{\partial u}{\partial x}\right)_{x=a} = 0$, $t > 0$ and $u(x,0)=x$, $0 \le x \le a$ is

$$u(x,t) = \frac{1}{2}a - \frac{4a}{\pi^2}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}\cos\frac{(2n+1)}{a}\pi x \; e^{-\left[(2n+1)^2 k\pi^2 t\right]/a^2}$$ (20 Marks)

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[3] Find the solution of the interior and exterior Dirichlet problem $\nabla^2 u = 0$ in R or outside R where R is a circular region of radius a and $u(a,\varphi) = f(\varphi)$, $u(r,\varphi + 2\pi) = u(r,\varphi)$ where f is a given function and φ is the angular coordinate , u remains bounded as $r \to \infty$. (20 Marks)

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[4] Using Laplace's transform to solve the equation $x\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = x$ $(x > 0, t > 0)$ given $u(x,0) = 0$, $x > 0$ and $u(0,t) = 0$, $t > 0$ (15 Marks)

Good luck **Dr. Mahasen Moussa**

المستوى : الرابع		دور مايو ٢٠١٥
الشعبة: رياضيات		الزمن: ساعتان
المادة : تحليل مركب (٢) ر ٤١٧	كلية العلوم – قسم الرياضيات	التاريخ: ٢٠١٥/٥/٢٣

Answer the following questions

[1]-a) Define pole of order m for $w = f(z)$ at $z = z_0$. If f(z) has a pole of order m at $z = z_0$. Then prove that

$$\text{Res}\,[f, z_0] = \lim_{z \to z_0} \frac{1}{(m-1)!} \frac{d^{(m-1)}}{dz^{(m-1)}} \left[(z - z_0)^m f(z) \right] \qquad \text{(10 Marks)}$$

b) Prove that $\int_{d}^{2\pi} \frac{d\theta}{1 + b \sin\theta} = \frac{2\Pi}{\sqrt{1 - b^2}} \left(|b| < 1 \right)$ (10 Marks)

[2]-a) Define zero of order m for $w = f(z)$. Let N and P are the number of zeros and poles of f(z).Then prove that

$$\frac{1}{2\Pi} \Delta_c \arg f(z) = N - P \qquad \text{(10 Marks)}$$

b) Desribe a Riemann surface for $w = z^{1/4}$. (10 Marks)

[3]- a) Under the transformation $w = \sqrt{2} e^{i\pi/4} z + (1 - 2i)$ find the image of $R : x = y = 0$, $x = 1$ and $y = 2$ in the w – plane. (10 Marks)

b) Prove that under $w = \frac{1}{z}$ straight lines and circles are mapped onto straight lines or circles. Find the image of $x + 3y - 2 = 0$ and $x - 3y + 2 = 0$ under $w = \frac{1}{z}$. (10 Marks)

[4]- a) Prove that $f(z) = \sin z$ is not bounded. Discuss the analyticity of $w = \cot z$. (3 Marks)

b) Prove that $\int_0^{\infty} \frac{\cos 2x}{x^2 + 1} dx = \frac{\pi}{2} e^{-2}$. (4 Marks)

c) Prove that the zeros of an analytic function are isolated. (3 Marks)

Good Luck **Prof. Dr. M. K. Aouf**

El-Mansoura- Egypt	Forth year of B.Sc.	المنصورة – مصر
Mansoura University	Program: B.Sc. (Statis.& Comp. Sc.) And (Math.)	جامعة المنصورة
Faculty of Science	Subject: Graph Theory	كلية العلوم
Mathematics Department	Course Code:	قسم الرياضيات
First Term: June 2015	**Date: 26 June. 2015**	**Time: 2 hours**

Answer the following five questions: **Mark**

1- a- Find the number of edges (or arcs) $|E(G)|$ of the graph G of each of:

(i) G is a disconnected graph with n vertices and maximal number of edges. **(2 points)**

(ii) A simple graph with n vertices and maximal number of edges. **(2 points)**

(iii) A simple graph G with 4 components, 10 vertices, and having a maximal number of edges. **(2 points)**

(iv) G is a bipartite graph with $2n$ vertices having a maximal number of edges . **(2 points)**

(v) G is a simple graph having maximum number of edges, $2n$ vertices and no triangles. **(2 points)**

b- Give an example of each of: (mention the reason (briefly)) :

(i) Two non-isomorphic simple graphs with 4 vertices and 4 edges. **(2points)**

(ii) A regular graph of order 2 with two components. **(2 points)**

(iii). A maximal planar graph with 5 vertices. **(2 points)**

(iv) Give an example of a nonplanar graph with 7 vertices. **(2 points)**

(v) An adjacency matrix of a graph with 4 vertices and 4 edges. **(2 points)**

2- Prove each of :

(i) Any connected finite graph G without circuits must have at least one vertices of degree 1 . **(5 points)**

(ii) If there is a direct circuit in a digraph, then there is a direct cycle **(5 points)**

(iii) The two graphs K_5 and $K_{3,3}$ are not planar. **(5 points)**

(iv) If G be a simple graph with n vertices and k components, then **(5 points)**

$$n - k \le |E(G)| \le \binom{n-k+1}{2} .$$

3- a- Let G be a connected graph. Show that: **(8 points)**

$$G \text{ is a tree} \Rightarrow |E(G)| = |V(G)| - 1$$

b- Give the definition of the isomorphism between two simple graphs. **(12 points)**
And give two non-isomorphic graphs with 7 vertices satisfying:
$\deg v_1 = \deg v_2 = 1$, $\deg v_3 = \deg v_4 = \deg v_5 = 2$, and $\deg v_6 = \deg v_7 = 4$..

4- a- *If G is a graph with $\deg v \ge 2$ for all $v \in V(G)$, then each component G_i has a cycle .* **(6 points)**

b- Let G be a plane graph with, n vertices , m edges and r regions. **(7 points)**
Mention and prove Euler formula for plane graphs?

c- Define what is rooted tree ? Let $T = (V, E)$ be a rooted tree with root v_0. **(7 points)**
Prove that indeg v =1 for all $v \in V - v_0$. And prove that T has no semi-cycles.

Examiner : Dr. Magdi H. Armanious **Full Mark: 80 points**

المستوى: الرابع الشعبة: رياضيات المادة: هيدروديناميكا	كلية العلوم- قسم الرياضيات	امتحان دور مايو ٢٠١٥ التاريخ: / /٢٠١٥ الزمن: ساعتان

Answer the following questions

[1]-a) State with proof the continuity equation.

b) The velocity and density fields in a flow are given by

$$q_x = kx^2y - \frac{\alpha}{y}, \quad q_y = -kxy^2 + \frac{\beta}{x}, \quad q_z = \varepsilon$$

and

$\rho = \rho_0 \exp[-\gamma(xy - ct)]$, where $\alpha, \beta, k, \varepsilon, \gamma$ and ρ_0 are constants, show that the flow satisfies the continuity equation if $c = \alpha + \beta$.

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[2]-a) Show that $\frac{D}{Dt} dq = d\frac{Dq}{Dt}$ and hence prove the constancy of circulation for an ideal flow.

b) Find the circulation of $\underline{q} = (x^2 - y^2)\underline{i} - 2xy\underline{j}$ around the circle $x^2 + y^2 = 1$.

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[3] Show that the combination of a uniform steam U and an apposite dipole of moment μ represent the flow around a circular cylinder. If a circulation k is added to the system find the positions of stagnation points.

مع أطيب الأمنيات بالنجاح والتوفيق — د/ محمود حمدى

Mansoura Univ.		4^{th} year math and Statistics	
Faculty of Science		Course Math. Model	
Mathematics Dept.		Date Jun.2015	
Subject:Math. R426	Time: 2 hours	80 marks	

1) i) Solve the following stochastic equation $dX=rXdt+sXdw$, where r,s are positive constants. Use it to explain the disappearance of some types of fish.

   ii) Solve the colonel Blotto's game. Comment on its real application.

   [27 marks]

2) i) Explain the epsilon constraint method to solve multi-objective optimization problem. Apply it to the following problem:
   min $f1=x-y$, $f2=2y-x$, $3\geqslant x,y\geqslant 0$.

   ii) Solve TSP with the following matrix [* 456, 3*45, 98*3, 969*].
   Discuss metaheuristic optimization.

   [26 marks]

3) i) Why fractals are abundant in nature. Derive the fractal dimension of the Cantor set.

   ii) Study SIS model on a static graph. Why do we need SWN.

   iii) Comment on the need for networks to study either cascade failure in power systems or economy.

   [27 marks]