دور مايو 2015 الزمن: ساعتان التاريخ: 2015/5/16	كليـــة العلوم – قسم الرياضيات	الفرقة: الرابعة الشعبة: رياضيات+إحصاء وعلوم الحاسب المادة: تحليل دالي

Answer the following questions

First : Objective questions : (20 marks)

Among the following statements mark the true and false ones with (√) and (×) respectively. Justify your answer for ONLY TWO of them :

(a) In a metric space X, for any set $A \sqsubseteq X$ we always have $\overline{\overline{A}} = A$. ....( )

(b) A complete metric space is a Banach space. ....( )

(c) Any subspace of a Banach space is also a Banach space. ....( )

(d) The sequence $\left(1/\sqrt{n}\right)$ belongs to the space ℓ^3. ....( )

(e) If $A = \{x_1, x_2, x_3, x_4, x_5, x_6\} \subseteq \ell^\infty$, it follows that the linear hull H(A) is separable. ....( )

(f) For any normed space E over K, the space L(ℓ^∞, E) is a Banach space. ....( )

(g)A normed space E that is linearly isometric to a Banach space F, is itself Banach space. ....( )

(h) For any normed space E over K, the dual space E^* is a Banach space. ....( )

(i) Every linear transformation $S: K^5 \to \ell^\infty$ is continuous on R^5. ....( )

(j) If E is a normed space over K, then any two norms defined on E are equivalent. ....( )

Second : Subjective questions (26 marks each)

[2] a. Define: a metric space - the space ℓ^p. (4 marks)

Let $p > 1$, $q = p/(p-1)$. Prove that if $(\alpha_n) \in \ell^p, (\beta_n) \in \ell^q$, then the series $\sum_n \alpha_n \beta_n$ is absolutely convergent ; and

$$\sum_{n=1}^{\infty} |\alpha_n \beta_n| \le \left(\sum_{n=1}^{\infty} |\alpha_n|^p\right)^{1/p} \cdot \left(\sum_{n=1}^{\infty} |\beta_n|^q\right)^{1/q}.$$ (10 marks)

b. Show that a convergent sequence in a metric space has a unique limit. (6 marks)

[3]] a. Define : a separable space. (2 marks)

Show that the space ℓ^p is separable ($p \ge 1$). (8 marks)

b.Let E,F be normed spaces over K , $E \neq \{0\}$, and let $T : E \to F$ be a linear mapping of E onto F. Prove that that T is 1-1 and T^{-1} is bounded <u>if and only if</u> there exists a constant $m > 0$ such that $\|Tx\| \ge m$,for all $x \in E$ with $\|x\| = 1$. (10 marks)

[4] a. Consider the mapping $T: R^2 \to R^3$, $T(a,b) = (2a+b, a-2b, b)$ for all $(a,b) \in R^2$

Show that T is a bounded linear transformation on R^2 and then find $\|T\|$. (9 marks)

b. Let E,F be normed spaces over K and suppose that E is finite-dimensional. Prove that :

(i) E is a Banach space , and (ii) every linear transformation $T: E \to F$ is continuous on E.

(3+8 marks)

Best wishes..Ever

Samir Kassem

MANSOURA UNIVERSITY

Algorithms Analysis and Design
Faculty of Science
Statistics and Computer Science

Final Exam
Date: May 19, 2015
Time: TWO Hours

Second Semester 2014/2015
Dr. Samir Elmougy

ANSWER the following FOUR Questions. NO MARKS without listing all steps.

Question 1 [Marks 15]

A. [5.0 Marks] What is big-O for the following loops (list all steps)

```
for ( int i = 0, i <= n, i++)
    for ( int j = 0, j <= n, j++)
        for ( int k = 0, k <= n^2, k++)
            sum++;
```

B. [5.0 Marks] Solve the following recurrence relation using recursion tree method:

$$T(n) = 2T(n/2) + c.n \text{ where } T(1) = c$$

C. [5.0 Marks] Discuss whether Master table could solve $T(n)=T(n-1)+7$, $T(1)=4$ or not.

Question 2 [Marks 20]

A. [10.0 Marks=5.0+5.0] Write Quick-Sort algorithm? Analyze its complexity in the best case scenario.

B. [4.0 Marks]Apply Quick-Sort sort algorithm to sort the following set of integers in increasing order: 12, 5, 17, 4, 6, 11, 10, 20, 5.

C. [3.0 Marks=1.0+2.0] Is Quick-Sort NP-problem? WHY?

D. [3.0 Marks=1.0+2.0] Is Insertion-Sort better than Heap-Sort? WHY?

Question 3 [Marks 15]

A. [3.0 Marks] What is Data Compression?

B. [5.0 Marks]Write Huffman codes algorithm to encode a text file.

C. [5.0 Marks] Analyze the complexity of Huffman codes algorithm.

D. [2.0 Marks] What is the type of design used in Huffman codes algorithm?

Question 4 [Marks 10 = 7+3]

Write an efficient algorithm to find the median of an unsorted set of n integer numbers. For example: if the set of numbers is 5, 3, 4, 7, 9, 20, 4, 13, 12 then the median is 7. What is the time complexity of your algorithm?

With my best wishes
Dr. *Samir Elmougy*

Mansoura university	2_nd term	المقرر: تحليل التباين (ر ٤٣٤)
Faculty of science	2014/2015	الزمن : ساعتان
Math. Depart	4^{th} year (stat., computer sci.)	التاريخ : ٢٠١٥/٥/٢٣
	Final exam	

Answer the following questions

Q1: (27 marks)

(a) Construct one way analysis of variance table and test the hypothesis $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$, at level of significance $\alpha = 0.o5$, for the following observations

Treatment	Observations					
A	7	10	9	8	10	10
B	5	6	4	8	7	6
C	8	9	10	6	3	6
D	5	6	4	3	7	5

(b)If H_0 is rejected, use Scheffe's test to compare between four population means.

(c) Compare between the two groups of treatments (A, B, C) versus (D) using the contrast $w = \mu_1 + \mu_2 + \mu_3 - 3\mu_4$.

Q2: 27 marks)

For the observations in the following table

treatments	observations					
A	4	7	6	3		
B	7	8	6	6	5	4
C	5	6	7			

(a) Test the hypothesis that $H_0: \mu_1 = \mu_2 = \mu_3$ at level of significance $\alpha = 0.05$.

(b) Test the homogeneity of variances at level of significance $\alpha = 0.01$.

Q3: (26 marks)

Suppose that we are interested in the yields of 3 varieties A, B and C of wheat using 4 different fertilizers, planted in 12 randomly selected pieces of land with the same fertility assumption, production was, as in the following table. The yields for the three varieties of wheat measured in 100 kg

	Fertilizers			
Wheat	I	II	III	IV
A	70	56	51	61
B	66	60	55	60
C	77	67	59	65

(a) Are there any differences between the impact of different types of fertilizers

(b) Are there any differences between the productions of different types of wheat?

[Use $\alpha = 0.05$].

Note that:

$b_3(0.01,4) = 0.3165$, $b_3(0.01,6) = 0.5149$, $b_3(0.01,3) = 0.1672$, $f_{0.05}(2,10) = 4.10$, $f_{0.05}(1,20) = 4.35$, $f_{0.05}(3,20) = 3.10$, $f_{0.05}(2,6) = 5.14$, $f_{0.05}(3,6) = 4.76$.

El-Mansoura- Egypt	Forth year of B.Sc.	المنصورة – مصر
Mansoura University	Program: B.Sc. (Statis.& Comp. Sc.) And (Math.)	جامعة المنصورة
Faculty of Science	Subject: Graph Theory	كلية العلوم
Mathematics Department	Course Code:	قسم الرياضيات
First Term: June 2015	**Date: 26 June. 2015**	**Time: 2 hours**

Answer the following five questions: **Mark**

1- a- Find the number of edges (or arcs) $|E(G)|$ of the graph G of each of:

(i) G is a disconnected graph with n vertices and maximal number of edges. **(2 points)**
(ii) A simple graph with n vertices and maximal number of edges. **(2 points)**
(iii) A simple graph G with 4 components, 10 vertices, and having a maximal number of edges. **(2 points)**
(iv) G is a bipartite graph with $2n$ vertices having a maximal number of edges . **(2 points)**
(v) G is a simple graph having maximum number of edges, $2n$ vertices and no triangles. **(2 points)**

b- Give an example of each of: (mention the reason (briefly)) :

(i) Two non-isomorphic simple graphs with 4 vertices and 4 edges. **(2points)**
(ii) A regular graph of order 2 with two components. **(2 points)**
(iii). A maximal planar graph with 5 vertices. **(2 points)**
(iv) Give an example of a nonplanar graph with 7 vertices. **(2 points)**
(v) An adjacency matrix of a graph with 4 vertices and 4 edges. **(2 points)**

2- Prove each of :

(i) Any connected finite graph G without circuits must have at least one vertices of degree 1 . **(5 points)**
(ii) If there is a direct circuit in a digraph, then there is a direct cycle **(5 points)**
(iii) The two graphs K_5 and $K_{3,3}$ are not planar. **(5 points)**
(iv) If G be a simple graph with n vertices and k components, then **(5 points)**

$$n - k \leq |E(G)| \leq \binom{n-k+1}{2} .$$

3- a- Let G be a connected graph. Show that: **(8 points)**

$$G \text{ is a tree} \Rightarrow |E(G)| = |V(G)| - 1$$

b- Give the definition of the isomorphism between two simple graphs. **(12 points)**
And give two non-isomorphic graphs with 7 vertices satisfying:
$\deg v_1 = \deg v_2 = 1$, $\deg v_3 = \deg v_4 = \deg v_5 = 2$, and $\deg v_6 = \deg v_7 = 4$..

4- a- *If G is a graph with* $\deg v \geq 2$ *for all* $v \in V(G)$, *then each component* G_i *has a cycle .* **(6 points)**

b- Let G be a plane graph with, n vertices , m edges and r regions. **(7 points)**
Mention and prove Euler formula for plane graphs?

c- Define what is rooted tree ? Let $T = (V, E)$ be a rooted tree with root v_0. **(7 points)**
Prove that indeg v =1 for all $v \in V - v_0$. And prove that T has no semi-cycles.

Examiner : Dr. Magdi H. Armanious **Full Mark: 80 points**

امتحان دور مايو 2015 م برنـامج : احصاء و علوم الحاسب المستوى: الرابع اسم المقرر : سلاسل زمنية و تنبؤ كـود المادة : ر 435		جامعة المنصورة – كلية العلوم قسـم الرياضيات التـاريخ : 30 / 5 / 2015 م الدرجة الكـلية : 80 درجة الزمن : سـاعتان

Answer the following questions:

[Q1] a- Let the causal process for a time series $\{X_t\}$, $t \in Z$ be given by

$$X_t - \varphi_1 X_{t-1} - \varphi_2 X_{t-2} = Z_t + \theta Z_{t-1} \quad \text{where} \{Z_t\} \sim WN(0, \sigma^2).$$

i) Write down the operator form of this process. Under what conditions would this be an ARMA(2, 1) process? **(4 marks)**

ii) Obtain the linear process form of this time series when $\varphi_1 = 0.8$, $\varphi_2 = 0.1$ and $\theta = 0.3$ **(11 marks)**

b- Consider the time series model $X_t = m_t + Y_t$, where m_t is a polynomial trend of degree three and Y_t denotes the random noise component. Find a five point filter $\{a_j\}_{j=-2,\cdots,2}$ **(15 marks)**

[Q2]a- Find the invertible representation of MA(1) process given by $X_t = Z_t + \theta Z_{t-1}$, where $\{Z_t\} \sim WN(0, \sigma^2)$. **(10 marks)**

b- Explain how the operators ∇ and ∇_d can be used to remove the trend and seasonality from the time series $\{X_t\}$, $t \in Z$ **(10 marks)**

c- Define 1) ARMA(p , q) process 2) white noise process 3) random walk 4) q- correlated time series 5) symmetric two sided moving average **(10 marks)**

[Q3]a- Define stationary and strictly stationary processes and show that stationarity does not imply strict stationarity. **(10 marks)**

b- Find $\rho(k)$ of the causal process $X_t - 0.7 X_{t-1} + 0.1 X_{t-2} = Z_t$ **(10 marks)**

Best wishes Dr. Faten Shiha

Mansoura Univ.	4^{th} year math and Statistics
Faculty of Science	Course Math. Model
Mathematics Dept.	Date Jun.2015
Subject:Math. R426 Time: 2 hours	80 marks

1) i) Solve the following stochastic equation dX=rXdt+sXdw, where r,s are positive constants. Use it to explain the disappearance of some types of fish.

   ii) Solve the colonel Blotto's game. Comment on its real application.

   [27 marks]

2) i) Explain the epsilon constraint method to solve multi-objective optimization problem. Apply it to the following problem:
   min f1=x-y, f2=2y-x, $3 \geqslant x, y \geqslant 0$.

   ii) Solve TSP with the following matrix [* 456, 3*45, 98*3, 969*]. Discuss metaheuristic optimization.

   [26 marks]

3) i) Why fractals are abundant in nature. Derive the fractal dimension of the Cantor set.

   ii) Study SIS model on a static graph. Why do we need SWN.

   iii) Comment on the need for networks to study either cascade failure in power systems or economy.

   [27 marks]