El-Mansoura- Egypt	4th level of Math. Prog. and Statis. and comp. Sc.	مصر – المنصورة
Mansoura University Faculty of Science	Program: M.Sc. (Statistics and Computer Science) Subject: Lattice Theory	جامعة المنصورة كلية العلوم
Mathematics Department First Term: Des. 2014	Course Code: Math. 412 Date: 27 Des. 2014 Time: 2 hours	قسم الرياضيات

Answer the following five questions:

1- Give two distinct equivalent definitions of meet semilattices, one as a poset and the other as an algebra.   (10 points)

And then give an example of each of: (each item 2 points)

     1- A partially ordered set (poset) but not a lattice.

     2- A non- modular lattice with 6 elements.

     3- A poset has more than one maximal element.

     4- A distributive lattice having more than 4 elements.

     5- An $\le$ - homomorphism between two lattices but not $\vee$- homomorphism.

2- a- Let N be the set of natural numbers. Prove that $(N ; \preccurlyeq)$ is a lattice

     where $\preccurlyeq$ defined by $x \preccurlyeq y :\Leftrightarrow x \mid y$. (Hint: determine the GLB(x, y)

     and the LUB(x, y) for each x , y $\in N$). Also, let N_{18} be the set of

     all devisors of 18, then give Hass Diagram of the lattice $(N_{18}, \preccurlyeq)$.   (10 points)

   b- Find all posets with 4-element set and determine,

     which one is a meet-simelattice, a join-simelattice, or a lattice.   (5 points)

   c- In a lattice (L, $\vee$, $\wedge$) prove that:

$$\text{If } a_1 \le b_1 \text{ \& } a_2 \le b_2 \Rightarrow a_1 \wedge a_2 \le b_1 \wedge b_2 .$$ (5 points)

3- a- Give two equivalent definitions of a $\vee$-ideal of a lattice   (10 points)

     $L = (L ; \vee , \wedge)$ and prove the equivalence between them.

   b - Define a congruence relation θ on a lattice $L = (L ; \vee , \wedge)$.   (10 points)

     And show that each congruence class $[a]\theta$ is a convex sublattice.

4- a- Let a , x, y be any three elements in a lattice $L = (L ; \vee , \wedge)$.   (10 points)

     Prove that:

$$L \text{ is distributive} \Leftrightarrow \text{“ } a \wedge x = a \wedge y \text{ \& } a \vee x = a \vee y \Rightarrow x = y \text{ “}.$$

   b- Let (L, $\vee$, $\wedge$) be a lattice. Prove that:

     (L, $\vee$, $\wedge$) is not modular lattice $\Leftrightarrow$ N_5 is a sublattice of (L, $\vee$, $\wedge$)   (10 points)

---+---

Examiner : Dr. Magdi H. Armanious   **Full Mark: 80 points**

Answer all questions:
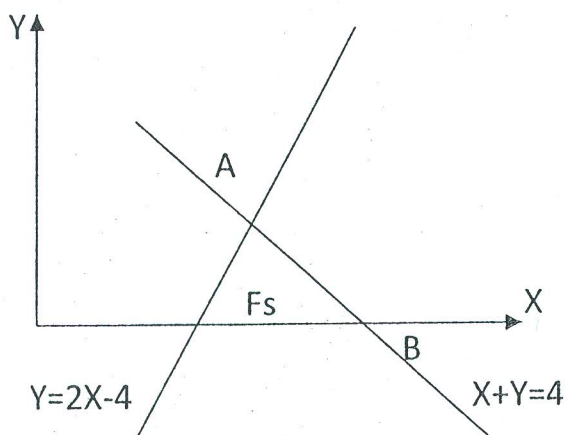
Question[1]

a- Define:

   (i)   Convex set   (ii)   Convex function   (iii)   Convex linear programming

   (iv)   Convex hull   (v)   Feasible solution   (vi)   Optimal feasible solution

b- By using the graphical method solve the LPP:

   Max Z=8x-4y   such that: (i) $|x+y| \leq 5$ (ii) $|x-y| \leq 5$

c- Consider the feasible region shown below:

   (i) Determine the coordinates of vertex B.

   (ii) Determine the coordinates of vertex A.

   (iii) Write the system of linear inequalities that formed the feasible region(Fs)

(الدرجة ٢٠)

Question[2]

a- Let S be a nonempty convex set in R^n and let $f : S \longrightarrow R$ be a convex function. Then, prove that the level set $S_\alpha = \{x \in S | f(x) \leq \alpha\}$, α is a real number, is a convex set?

b- By using the simplex method solve the LPP:

   Max   Z=5x_1+4x_2

   such that: $4x_1 + 5x_2 \leq 10$, $3x_1 + 2x_2 \leq 9$, $8x_1 + 3x_2 \leq 12$, $x_1, x_2 \geq 0$

c- Show that: $f(x) = 3x + 4$ $\forall$ $x \in X \subset R^n$ is a convex function?

(الدرجة ٣٠)

(من فضلك اقلب الورقة)

Question[3]

a- True or false:

    (i)   The union of two convex sets is convex set.

    (ii)   If $f : R^n \longrightarrow R$ be a concave function over a convex set S then $\dfrac{1}{f(x)}, f < 0$ is

          concave function.

    (iii) The extreme points of the set $\{(x,y): |x| \le 1, \ |y| \le 1\}$ are $\{(1,-1),(1,1),(-1,1),(-1,-1)\}$

    (iv) Minimize Z=-Maximize {-Z}

b- Solve the following transportation problem using the North-West corner method:

	D1	D2	D3	D4	Availability
O1	6	4	1	5	14
O2	8	9	2	7	16
O3	4	3	6	2	5
Requirement	6	10	15	4	

c- Let S_1 and S_2 be convex sets in R^n. prove that:

    (i) $S_1 - S_2$ is convex set

    (ii) $S_1 + S_2$ is convex set

<div dir="rtl">(الدرجة ٣٠)</div>

<div dir="rtl">مع تمنياتى بالنجاح والتفوق
د.محمد عبد الرحمن</div>

4th Level Exam
Mathematics
Statistics & Computer Science

Numerical Analysis

Faculty of Science
Mathematics Department

January **2014**

Time : **2 hours**

Full mark **60**

Answer the following questions

[1] a) Find the first two iterations of the Gauss-Seidel method for the following system , using $X^{(0)} = (0,0,0)^t$

$$x_1 + 2x_2 + 5x_3 = -3, \quad 3x_1 + x_2 + 4x_3 = -2, \quad 6x_1 + 2x_3 = 2 \qquad \text{(7 marks)}$$

b) Find the least squares polynomials of degree 1 for the data in the following table.

x_i	1.0	1.1	1.3	1.5	1.9	2.1
y_i	1.84	1.96	2.21	2.45	2.94	3.18

(7 marks)

[2] a) Let $T_n(x)$ denote the Chebyshev polynomial, show that :

(i) $T_{n+1}(x) = 2x\, T_n(x) - T_{n-1}(x)$,

(ii) $\int_{-1}^{1} \frac{T^2{}_n(x)}{\sqrt{1-x^2}}\, dx = \frac{\pi}{2}$.   (8 marks)

b) Show that the function $G : D \subset \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ has a fixed point in D where

$$G(x_1, x_2, x_3) = \left(\frac{1}{3}\cos(x_2 x_3) + \frac{1}{6},\ \frac{1}{9}\sqrt{x^2{}_1 + \sin x_3 + 1.06} - 0.1,\ \frac{-1}{20}e^{-x_1 x_2} - \frac{10\pi - 3}{60} \right),$$

$$D = \{ (x_1, x_2, x_3) : -1 \le x_i \le 1, \quad i = 1, 2, 3 \}. \qquad \text{(8 marks)}$$

[3] a) Find the first two iterations of the Newton's method for the following nonlinear system using $X^{(0)} = (0,0)^t$

$$x^2{}_1 - 10x_1 + x^2{}_2 + 8 = 0, \quad x_1 x^2{}_2 + x_1 - 10x_2 + 8 = 0 . \qquad \text{(8 marks)}$$

b) Find the general continuous least squares trigonometric polynomial Sn(x) for

$$f(x) = \begin{cases} -1, & \text{if } -\pi < x < 0, \\ 1, & \text{if } 0 < x < \pi. \end{cases} \qquad \text{(7 marks)}$$

[4] a) Determine the Pad'e approximation of degree 5 with n = 2 and m = 3 for $f(x) = e^x$.

(7 marks)

b) Find the first two iterations obtained by the power method with scaling and hence approximate the dominant eigenvalue of the following matrix

$$\begin{pmatrix} 1 & 2 & 0 \\ -2 & 1 & 2 \\ 1 & 3 & 1 \end{pmatrix}, \quad \text{using } X^{(0)} = (1,1,1)^t \ . \qquad \text{(8 marks)}$$

Best Wishes

Dr. Mahmoud Abdelaziz Elbiomy

عن الأسئلة الآتية :- السؤال الاول: (20 درجة)(8 , 6 , 6 درجات)

1) عرف كلاً من : الخطأ من النوع الأول (α) و الخطأ من النوع الثانى (β) ثم قارن بينهما .

2) إذا كانت P تمثل نسبة الأصوات التى اختارت الشخص A ضد الشخص B فى انتخابات ما . أخذت عينة حجمها 20 شخص و كان X يمثل عدد الأشخاص الذين يؤيدون الشخص A فى العينة :

ا) احسب α حيث $H_0 : P = 0.5$ مقابل $H_1 : P \neq 0.5$ و منطقة الرفض $X \leq 3$ أو $X \geq 15$

ب) احسب الخطأ من النوع الثانى β حيث $H_1 : P = 0.9$ على نفس منطقة الرفض السابقة .

السؤال الثانى: (30 درجة كل جزء 10 درجات)

1) تكلم عن اختبار الاشارة ثم اشرح كيفية اختبار عشوائية عينة ما ؟

2) اشرح بالتفصيل خطوات اختبار فرض احصائى حول الفرق بين وسطى مجتمعين مستقلين او غير مستقلين

3) عرف كلا من : Z_α , $Z_{0.025}$, $t_{(0.025 , 10)}$, $\chi^2_{(0.05 , 8)}$, χ^2_α

السؤال الثالث: (30 درجة كل جزء 15 درجات)

الجدول التكرارى التالى يوضح توزيع أطوال عينة من النباتات بعد شهر من زراعتها :

اطوال النباتات	2 -	4 -	6 -	8 -	10 -	12 — 14
التكرار المشاهد	13	17	18	20	18	14

1) هل أطوال النبات تتبع توزيع طبيعى ؟ استخدم مستوى معنوية $\alpha = 0.025$

2) اختبر الفرض $H_0 : \mu = 8$ & $H_1 : \mu \neq 8$ على مستوى معنوية $\alpha = 0.05$

$Z_{0.05} = 1.645$, $Z_{0.025} = 1.96$, $t_{(0.025 , 10)} = 2.23$, $t_{(0.025 , 9)} = 2.26$,

$\chi^2_{(0.025 , 3)} = 9.35$, $\chi^2_{(0.975 , 3)} = 0.22$, $\chi^2_{(0.025 , 2)} = 7.38$, $\chi^2_{(0.975 , 2)} = 0.05$

(اقلب الورقة)

مع أطيب التمنيات بالتوفيق
د. محمـد جــاد

المساحة المظللة تمثل $P(0 < Z < z)$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2703	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997

Answer the following questions:

Question #1 [30 Marks]

a) What are the advantages of neural networks?

b) State the main characteristics of neural networks?

c) What is the difference between:

1. Adaptation and generalization

2. Digital computers and neural networks

3. Supervised and unsupervised learning

d) What is the purpose of using activation function? State its different types?

e) Draw the diagram of:

1. Hopfield neural model

2. Neural networks used in computer vision

f) An odd sigmoid is function by:

$$\Phi(x) = \frac{1 - e^{-ax}}{1 + e^{-ax}}$$

The limiting values of this function are -1 and +1. Show that the derivative of $\Phi(x)$ with respect to x is given by:

$$\frac{\partial \Phi}{\partial x} = \frac{a}{2}[1 - \Phi^2(x)]$$

What is the value of this derivative at the origin? Suppose that the slope parameter a is made infinitely large. What is the resulting form $\Phi(x)$?

g) Determine the storage capacity of a Hopfield network with 64 neurons used to store the following:

       1. Binary patterns   2. Bipolar patterns

Question #2 [30 Marks]

a) State some applications of neural networks?

b) How can the optimal network architecture be decided?

c) A neuron j receives inputs from four other neurons whose activity levels are 10,-20, 4, and -2. The respective weighs of neuron are 0.8, 0.2,-1.0 and -0.9. Calculate the output of neuron j for the following two situations:

1. The neuron is linear.

2. The neuron is represented by a binary step activation function.

Assume that the bias applied to the neuron is zero.

d) Construct an architecture graph of:

1. A fully connected feedforward network has 10 source nodes, 2 hidden layers, one with 4 neurons and the other with 3 neurons, and a single output neuron.

2. Feedback neural network has 4 source nodes, 4 hidden neurons, and 4 output neurons.

e) Design a neural network to perform the following logic operations AND, NOR, XOR?

f) Realize the following equation by using neural networks:

$$Z = \frac{4}{\sqrt{12X + 4Y}} - \frac{9}{(6X - 3Y)^2}$$

Draw the architecture of the network and state the values of weights and activation functions?

g) Write a MATLAB program to generate a few activation functions that are being used in neural networks?

Mansoura University Faculty of Sciences Mathematics Dept. Neural Networks		4th year Time: 2 Hours Date: 13/1/2015 Maximum 60 Marks

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Page 1/2

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Final Exam
Data Structures – Math 441
4th Level
Mathematics Department
Faculty of Sciences
Mansoura University
Questions in 1 Page – Total Marks: 60

Duration: 2 Hours	Answer **ALL** the Questions	10th January 2015

Question One: Mark each sentence with True or False. Use Question Number Only. No Need to copy the statement into your answer paper. Write the Correct statement for false ones.............. **(20 Marks)**

1. Arrays is a collection to store multiple objects.
2. Arrays can include different types of objects.
3. Arrays is sequential based access collection.
4. Multidimensional array is called Matrix.
5. Linked list is a random access collection.
6. In Singly linked list, there are two pointers in each node.
7. In Doubly linked list, there are two pointers in each node.
8. We can traverse Singly linked list starting from tail to head.
9. We can traverse Doubly linked list starting from tail to head.
10. Stack is FIFO data structure.
11. Queue is a FIFO data structure.
12. Circular array is used to overcome challenges of arrays.
13. Algorithm analysis is the science of identifying the complexity of algorithms.
14. We must implement algorithm using programming language to identify its complexity.
15. In linear search, Best Case happens when we do not find the element we are searching for.
16. Linear Search complexity is $O(n^2)$.
17. Binary Search complexity if $O(n)$.
18. Recursion is the capability of a function/module to call itself.
19. Recursion always returns an answer.
20. Compilers can execute pseudo-code.

Question Two: .... **(40 Marks)**

1. Write an application to store a sequence of high score entries for a video game. Define the required Classes to build the application.
2. Illustrate and Write the application of Tic-Tac-Toe showing how multidimensional arrays can be used in game programming.
3. Compare with figure and Java code between Singly and Doubly linked lists, highlighting advantages and disadvantages of each of them.
4. What are the required operations to insert values at the head, the tail, and in the middle of Singly linked list. Illustrate with figure.
5. Compare between Stack and Queue. Mention some real world examples of using both of them in Computer Applications.
6. Illustrate with figure how Circular Arrays work, and why we need them.
7. Define with drawing the functions of BigO Notation used in algorithm complexity analysis.
8. Compare between Binary Search and Linear Search.
9. Define Recursion, and illustrate its main components.
10. Using Recursion, write the Java code needed to calculate the sum of array elements.

Best wishes,
Dr.Haitham A. El-Ghareeb