/ ٤١٨ نظرية الشبكات

El-Mansoura- Egypt	4^{th} level of Math. Prog. and Statis. and comp. Sc.	مصر - المنصورة
Mansoura University Faculty of Science	Program: M.Sc. (Statistics and Computer Science) Subject: Lattice Theory	جامعة المنصورة كلية العلوم
Mathematics Department **First Term: Des. 2014**	Course Code: Math. 412 **Date: 27 Des. 2014**	قسم الرياضيات **Time: 2 hours**

Answer the following five questions:

1- Give two distinct equivalent definitions of meet semilattices, one as a poset and the other as an algebra. (10 points)

And then give an example of each of: (each item 2 points)

1- A partially ordered set (poset) but not a lattice.
2- A non- modular lattice with 6 elements.
3- A poset has more than one maximal element.
4- A distributive lattice having more than 4 elements.
5- An $\leq$ - homomorphism between two lattices but not $\vee$- homomorphism.

2- a- Let N be the set of natural numbers. Prove that (N ; $\preccurlyeq$) is a lattice where $\preccurlyeq$ defined by $x \preccurlyeq y :\Leftrightarrow x \mid y$. (Hint: determine the GLB(x, y) and the LUB(x, y) for each x , y $\in$ N). Also, let N_{18} be the set of all devisors of 18, then give Hass Diagram of the lattice (N_{18}, $\preccurlyeq$). (10 points)

b- Find all posets with 4-element set and determine, which one is a meet-simelattice, a join-simelattice, or a lattice. (5 points)

c- In a lattice (L, $\vee$, $\wedge$) prove that:

If $a_1 \leq b_1$ & $a_2 \leq b_2 \Rightarrow a_1 \wedge a_2 \leq b_1 \wedge b_2$. (5 points)

3- a- Give two equivalent definitions of a $\vee$-ideal of a lattice L = (L ; $\vee$, $\wedge$) and prove the equivalence between them. (10 points)

b - Define a congruence relation θ on a lattice L = (L ; $\vee$, $\wedge$). (10 points)
And show that each congruence class $[a]\theta$ is a convex sublattice.

4- a- Let a , x, y be any three elements in a lattice L = (L ; $\vee$, $\wedge$). (10 points)
Prove that:

L is distributive $\Leftrightarrow$ " $a \wedge x = a \wedge y$ & $a \vee x = a \vee y \Rightarrow x = y$ ".

b- Let (L, $\vee$, $\wedge$) be a lattice. Prove that:

(L, $\vee$, $\wedge$) is not modular lattice $\Leftrightarrow$ $\mathbf{N_5}$ is a sublattice of (L, $\vee$, $\wedge$) (10 points)

Examiner : Dr. Magdi H. Armanious **Full Mark: 80 points**

دور: يناير 2015 الزمن : ساعتان التاريخ : 30 /2014/12	كلية العلوم – قسم الرياضيات	الفرقة: المستوى الرابع المادة : هندسة تفاضلية كود المادة : ر416 البرنامج : رياضيات

Answer the following questions: *Full mark : 80*

1-a) Define: the regular curve , the arc- length function and show that the curve $\alpha(\theta) = (1+\cos\theta \;, \sin\theta, \; 2\sin\frac{\theta}{2})$ is regular and lies on a sphere centered at the origin with radius 2 and find its speed and the velocity vector

b) Deduce the expression of the second fundamental form II of a surface and prove that II of a plane is zero .

2- a) For a space curve $\alpha(s)$,define B , the torsion τ and prove that $\frac{dB}{ds}$ is parallel to N and find Frenet formulae.

b) Define the Gaussian curvature K and the normal curvature K_n of a regular surface . And find them for the surface

$$r(\theta, \varphi) = ((b+a\sin\varphi)\cos\theta, \; (b+a\sin\varphi)\sin\theta \;, a\cos\varphi)$$

3- a) Define T , N , the curvature k of a curve $\alpha(t)$ and show that

$$N = \frac{dT}{dt} / \left|\frac{dT}{dt}\right| \;, \quad K = \frac{|\dot{\alpha} \times \ddot{\alpha}|}{|\dot{\alpha}|^3} \;.$$

b) Define the regular surface S , the unit normal surface N^* and the tangent plane $T_p(S)$. Find the equation of $T_p(S)$ for the surface

$$z = \frac{1}{2}(x^2+y^2) \;, \quad \text{at } p = (1, -1, 1)$$

4- a) Prove that the curve $\alpha(t) = (e^t\cos t \;, e^t\sin t)$ is regular and find T , N for it .

b) Compute the curvature k and the torsion τ of the curve

$$\alpha(t) = (\frac{4}{5}\cos t \;, 1-\sin t \;, -\frac{3}{5}\cos t).$$

Best wishes.

٤ رياضيات

دور يناير ٢٠١٥ الزمن: ساعتان التاريخ:22/1/2015	كلية العلوم- قسم الرياضيات	الفرقة: الرابعة الشعبة: ر+ح ص المادة: بحوث عمليات(ر ٤٢١)

Answer all questions:

Question[1]

a- Define:

(i) Convex set (ii) Convex function (iii) Convex linear programming

(iv) Convex hull (v) Feasible solution (vi) Optimal feasible solution

b- By using the graphical method solve the LPP:

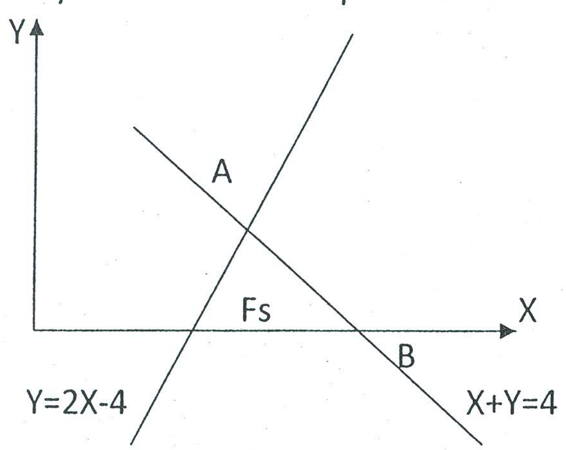
Max Z=8x-4y such that: (i) $|x+y| \leq 5$ (ii) $|x-y| \leq 5$

c- Consider the feasible region shown below:

(i) Determine the coordinates of vertex B.

(ii) Determine the coordinates of vertex A.

(iii)Write the system of linear inequalities that formed the feasible region(Fs)



(الدرجة ٢٠)

Question[2]

a- Let S be a nonempty convex set in R^n and let $f: S \longrightarrow R$ be a convex function. Then, prove that the level set $S_\alpha = \{x \in S | f(x) \leq \alpha\}$, α is a real number, is a convex set?

b- By using the simplex method solve the LPP:

Max $Z=5x_1+4x_2$

such that: $4x_1+5x_2 \leq 10$, $3x_1+2x_2 \leq 9$, $8x_1+3x_2 \leq 12$, $x_1, x_2 \geq 0$

c- Show that: $f(x)=3x+4 \quad \forall \quad x \in X \subset R^n$ is a convex function?

(الدرجة ٣٠)

(من فضلك اقلب الورقة)

Question[3]

a- True or false:

(i) The union of two convex sets is convex set.

(ii) If $f: R^n \longrightarrow R$ be a concave function over a convex set S then $\frac{1}{f(x)}, f<0$ is concave function.

(iii) The extreme points of the set $\{(x,y): |x| \le 1,\ |y| \le 1\}$ are {(1,-1),(1,1),(-1,1),(-1,-1)}

(iv) Minimize Z=-Maximize {-Z}

b- Solve the following transportation problem using the North-West corner method:

	D1	D2	D3	D4	**Availability**
O1	6	4	1	5	14
O2	8	9	2	7	16
O3	4	3	6	2	5
Requirement	6	10	15	4	

c- Let S_1 and S_2 be convex sets in R^n. prove that:

(i) $S_1 - S_2$ is convex set

(ii) $S_1 + S_2$ is convex set

(الدرجة ٣٠)

مع تمنياتى بالنجاح والتفوق

د.محمد عبد الرحمن

دور يناير ٢٠١٥ الزمن : ساعتين التاريخ ٢٠١٥/١/١٣	كلية العلوم قسم الرياضيات	المستوى : الرابع الشعبة : رياضيات المادة : زمر لى ر٤١٥

Answer the following questions

[1] Define what we mean by G is a topological group. Prove that every abstract group S is a topological group. Put true or false " every topological group is a Lie group and every Lie Group is a topological group.

[2] Define what we mean by a topological manifold of dimension m.
Prove that the graph of $y = x^{2/3}$ in R^2 is a topological manifold
Prove that the cross in R^2 is not a topological manifold.

[3] Define what we mean by G is a Lie group. Prove that $S^1 = \{z \in \mathbb{C} : |Z| = 1\}$ is a Lie group. Prove that $SY(2) = \{A \in GL(2,C); AA^{-t} = 1\}$ is a Lie Group.

[4] Define the algebraic structure between Lie Groups and Lie algebras
If $\gamma_x(t)$ is the one – parameter subgroup
with tangent vector at 1 equal to x then $\exp : g \to G$, by $\exp(x) = \gamma_x(1)$.
find the corresponding algebraic structure
of $G = R_1$ $g = R$
find the exp map of the lie group

$$G = S^1 = R/Z = \{Z \in \mathbb{C} : |Z| = 1\}$$

$$Z = e^{2\pi i\theta} \quad , \theta = R/Z$$

Good Luck **Prof. A. S. Hegazi**

٤ رياضيات - نظرية المعادلات التفاضلية (ر ٢)

Faculty of science
Math-department .

theory of differential equations
B.sc. Exam

January 2015
Time: 2 H

Answer the following questions :

1-a) State and prove the theorem of dependence of solutions on slight change on the initial conditions . (10marks)

b) Discuss the existence and uniqueness solution of the initial value problem

$\frac{dy}{dx} = y^2$, $y(1) = -1$, $R := \{ |x-1| \le a \ , \ |y+1| \le b \}$. Give the solution (10marks)

2-a) Give the orthonormal functions of the boundary value problem ,

$$\frac{d^2 y}{dx^2} + \lambda y = 0 \ , \ y(0) = 0 \ , \ y(\pi) = 0$$ (12marks)

b) Prove that a necessary and sufficient condition that the differential equation

$a_0(t)\frac{d^2x}{dt^2} + a_1(t)\frac{dx}{dt} + a_2(t)x = 0$ be a self-adjoin is that $\frac{d\,a_{0(t)}}{dt} = a_{1(t)}$ on $I = [a, b]$. (8marks)

3-a) If the vector functions $\varphi_1, \varphi_2, \ldots, \varphi_n$ defined by $\varphi_1 = \begin{pmatrix} \varphi_{11} \\ \varphi_{21} \\ \vdots \\ \varphi_{n1} \end{pmatrix}$, $\varphi_2 = \begin{pmatrix} \varphi_{12} \\ \varphi_{22} \\ \vdots \\ \varphi_{n2} \end{pmatrix}$,.... $\varphi_n = \begin{pmatrix} \varphi_{1n} \\ \varphi_{2n} \\ \vdots \\ \varphi_{nn} \end{pmatrix}$

are linearly <u>dependent</u> solutions of any H.L.V.D.E and $c_1, c_2, \ldots, c_n$ *are n costant* on $I = [a,b]$. Prove that $\varphi = c_1\varphi_1 + \cdots + c_n\varphi_n$ is a solution of the H.L.V.D.E , and the Wronskian ,
$W(\varphi_1, \varphi_2, \ldots, \varphi_n)(t) = 0$ (12marks)

b) For the existence and uniqueness theorem for the initial value problem $y' = f(x,y)$, $y(x_0) = y_0$, prove that the theory could be applied to over a large interval that granted by the(E&U) theorem .i.e there exist a unique solution over the open interval $a < x < b$,(Discuss and prove) . (8marks)

4-a) Prove that the unique solution φ of the H.L.V.D.E. $dx/dt = A(t)\,x$ that satisfies the I.C $\varphi(t_0) = x_0$, $t_0 \in I$ Can be expressed in the forms $\varphi(t) = \Phi(t)\,C$ and $\varphi(t) = \Phi(t)\,\Phi^{-1}(t_0)\,x_0$ where $\Phi(t)$ is a fundamental matrix . (10marks)

b) Give the solution of the L.H. V.D.E $$\frac{dx}{dt} = \begin{pmatrix} 7 & -1 & 6 \\ -10 & 4 & -12 \\ -2 & 1 & -1 \end{pmatrix} X$$. (10mark