جامعة المنصورة　　　　الفرقة الرابعة علوم　　　　الفصل الدراسي الاول 15-16

كلية العلوم دمياط　　　　برنامج الرياضيات وبرنامج الاحصاء وعلوم الحاسب　　　　التاريخ : 2 / 1 / 2016

قسم الرياضيات　　　　المادة :نظرية الشبكات (Lattice Theorey, Math 418)　　　　الزمن : 2 ساعات

Answer the following four questions:

1- 1- Give two distinct equivalent definitions of a lattice. (5 points)

And then give an example of each of:

(i) A partially ordered set but not a lattice, (3 points)

(ii) Find all posets with 4 elements and determine which one is a meet-lattece, a join-semilattice or lattice. (3 points)

(iii) A poset has more than one maximal element. (3 points)

(iv) A poset has only one maximal element. (3 points)

(v) All lattices of order 5. (3 points)

2- In a lattice $L(\vee, \wedge)$ prove that:

(i) There is always at least one maximal element in a finite poset. (5 points)

(ii) If ρ is a poset , then ρ^{-1} is also a poset. (5 points)

(iii) If $a_1 \leq b_1$ and $a_2 \leq b_2$, then $a_1 \vee a_2 \leq b_1 \vee b_2$. (5 points)

(iv) If f *is a* $\vee$-homomorphism , then f is a $\leq$-homomophism. (5 points)

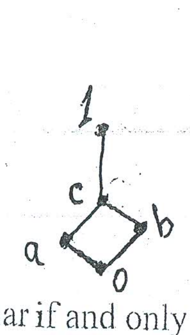
3- a- Give two equivalent definitions of ideals of a lattice $(L, ; \vee , \wedge)$ (10 points)

And prove the equivalence between them.

b- Define the principle ideal $a/0$ for $a \in L$,

and prove that $a/0$ is an ideal of L. (5 points)

c- determine the lattice of ideals of the lattice (5 points)



4- Prove that : The lattice $L = (L ; \vee , \wedge)$ is not modular if and only if L contains N_5 as a sublattice. And then give an example of a not modular lattice of order 6 (20 points)

Full marks 80 points

مع التوفيق د. مجدى حكيم

دور يناير ٢٠١٦ الزمن: ساعتان التاريخ: 5/1/2016	كلية العلوم- قسم الرياضيات	الفرقة: الرابعة الشعبة: ر+ح ص المادة: بحوث عمليات (ر ٤٢١)

Answer all questions:

Question[1]

a- Define:

(i) The convex set (ii) The convex function (iii) The feasible region

b- Choose the correct answer:

1- Which are the non-negative constraints of a linear program?

a. $X_1, X_2 > 0$ b. $X_1, X_2 >= 0$ c. $X_1 + X_2 > 0$ d. $X_1 + X_2 >= 0$

2- Equation $X_1 + 2X_2 = 5$ corresponds to a ___________ in a two-dimensional coordinate system.

a. point b. straight line c. circle d. parabola

3- A constraint must be an inequality (either "<=" or ">="). A constraint cannot be an equation ("=").

a. True b. False

4- A solution is feasible if it makes___________

a. all the constraints hold. b. at least one constraint hold

c- Show that $f(x) = |x| \quad \forall \ x \in R$ is a convex function?

(الدرجة ٢٠)

Question[2]

a- True or false :

(i) The union of two convex sets is convex set.

(ii) If $f : R^n \longrightarrow R$ be a concave function over a convex set S then $\frac{-1}{f(x)}, f < 0$ is concave function.

(iii) The extreme points of the set $\{(x,y): |x| \le 1, \ |y| \le 1\}$ are $\{(1,-1),(1,1),(-1,1),(-1,-1)\}$

(iv) Minimize Z=Maximize {-Z}

(v) The dual of the dual is primal

(vi) If f_1 and f_2 are convex functions then $f_1 + f_2$ is also convex

b- By using the Simplex method solve the problem:

$$\text{Max } Z = 2x_1 + 3x_2 \quad \text{Such that.}$$

$$2x_1 + 3x_2 \le 1, \quad 8x_1 + 2x_2 \le 3, \quad x_1, \ x_2 \ge 0$$

(اقلب الصفحة)

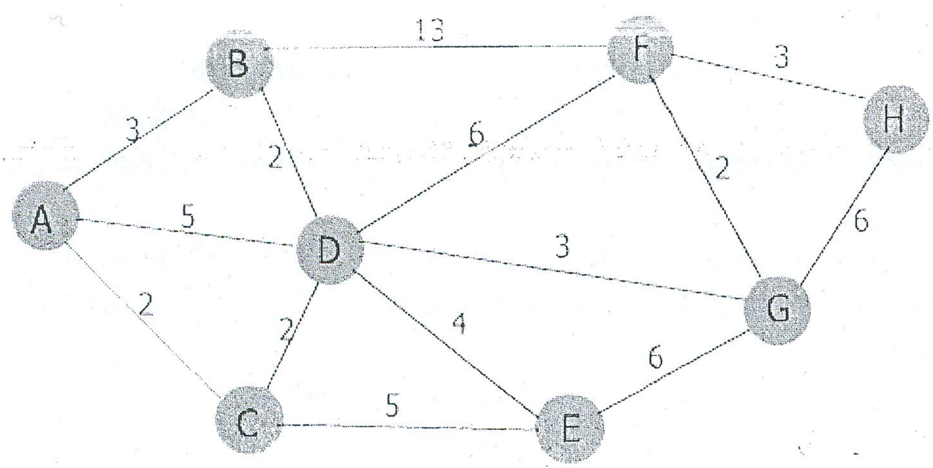
c- **Solve the following transportation problem using the Vogel method:**

	D1	D2	D3	D4	Availability
O1	6	4	1	5	14
O2	8	9	2	7	16
O3	4	3	6	2	5
Requirement	6	10	15	4	

(الدرجة ٣٠)

Question[3]

a- **Solve the minimum- spanning problem for given network:**



b- **For the following linear program, find the dual and solve it graphically:**

Min $Z = 2x_1 + 3x_2$ **Such that.** $2x_1 + 3x_2 \geq 1$, $8x_1 + 2x_2 \geq 3$, x_1 , $x_2 \geq 0$

(الدرجة ٣٠)

مع تمنياتى بالنجاح والتفوق   د.محمد عبد الرحمن

Mansoura Univ.
Faculty of Science
Mathematics Dept.
Subject:Math.
Course Quantum Mechanics (٤٣١)

4thYear: math.
Date Jan.2016
Time: 2 hours
Full marks: 80

Answer the following questions:

[1] a) Why classical mechanics is not suitable to study small particles (which are affected by emission or absorption of light)? Derive Schrodinger equation.

b) Derive Ehrenfest theorem. Explain its importance. [27 marks].

[2] Solve Schrodinger equation for the Hydrogen atom. [27 marks].

[3] a) Write short notice on quantum entanglement. Comment on Einstein objections to quantum mechanics.

b) Prove that the wave function sech(x) represents bound state .

c) Derive uncertainty principle and explain its relevance to the harmonic oscillator. [26 marks].

Mansoura University, Faculty of Science, Mathematics Department

امتحان مقرر تحليل عددي (2)- ر413 لطلاب السنة الرابعة بكلية العلوم شعبتي الإحصاء وعلوم الحاسب + الرياضيات
الفصل الدراسي الأول يناير 2016
Time Allowed: 2 hours

مسموح باستخدام الآلة الحاسبة . Answer the following questions . All questions carry equal marks

Question (1)

1-a) Describe the Butcher's table for the 4^{th} order Runge-Kutta (RK4) method that uses 4 stages per step.
1-b) Express the Chebyshev polynomial of the first kind , $T_n(x)$, n=1,2,... as a tri-diagonal determinant. How many real zeros in the open interval]-1,1[for $T_n(x)$?. Find these zeros.

1-c) Apply the Adams-Moulton method to approximate y at x=0.4 given that $\frac{dy}{dx} = y^2 + 4,\ y(0) = 0.$

Question (2)

2-a) Find the root between 0 and 1 of the equation $f(x) = x^3 - 6x + 4 = 0$ correct to four decimal places.
2-b) In some determination of the volume v of carbondioxide dissolved in a given volume of water at temperature θ, the following pairs of values were obtained

v	0	5	10	15
θ	1.8	1.45	1.8	1.00

Obtain by the method of least-squares a relation of the form $v = a + b\,\theta$.

Question (3)

3-a) Use the RK4 method to approximate y and z when x=0.1 for that particular solution of the system:

$$\begin{cases} \frac{dy}{dx} = z = f(x,y,z) \\ \frac{dz}{dx} = 4y - 2xz = g(x,y,z) \end{cases}$$

satisfying y =0.2, z=0.5 when x=0.
3-b) Find a quadratic least-squares approximation of the form : $p_2(x) = a_0T_0(x) + a_1T_1(x) + a_2T_2(x)$

For the function $f(x) = (1+x)^{\frac{1}{2}}, \quad -1 \le x \le 1$. Compute $\left| p_2(0) - f(0) \right|$.

Kind regards

Prof. Dr. Moawwad El-Mikkawy

Faculty of science
Math-department

theory of differential equations
B.sc. Exam

January : 2016
Time : 2 H

Answer the following question :

1a) Prove that the function $f(x, y) = x \sin y + y \cos x$ satisfies Liptischitz condition w.r. to y in $D := \{(x, y) : |x| \le a , |x| \le b\}$. (5marks)

b) Prove that for the I.V .problem $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$ their exist a unique solution $\varphi(t)$ on the domain D:= $a < x < b$, $-\infty < y < \infty$. (10marks)

c) Give the solution of the I.V . problem $\frac{dy}{dx} = x^3 + y^3 + x + 1$, $y(0) = 0$. (5marks)

2-a) For the H.L.V.D.E $\frac{dy}{dx} = \begin{pmatrix} 8 & 12 & -2 \\ -3 & -4 & 1 \\ -1 & -2 & 2 \end{pmatrix} X$ and its fundamental matrix $\Phi(t)$ .Give the unique solution $\varphi(t) = \Phi(t)\, \Phi^{-1}(t_o)\, x_o$ where $\varphi(t_o) = x_o = \begin{pmatrix} -1 \\ 4 \\ 2 \end{pmatrix}$, $t_o \in I$ any point you can choose it . (16marks)

b) Transform the L.D.E. $a_0(t) \frac{d^2y}{dt^2} + a_1(t) \frac{dy}{dt} + a_2(t)\, y = 0$ into the self-adgoint D.E.

$\frac{d}{dt}\left[P(t) \frac{dy}{dt} \right] + Q(t)\, y = 0$. (4marks)

3-a) Define the following : orthonormal functions , self-adjoint differential equation and discuss the limit function of a sequence of continuous real valued functions is continuous function ?? . (8marks)

b) Discuss the existence and uniqueness solution of any n^{th} order differential equation and for any system . (8marks)

c) Discuss the existence and uniqueness solution of the initial value problem

$\left(x^2 - \frac{1}{4} x - \frac{1}{8} \right) y'' + \frac{1}{x-3} y' + y = 0$, $y(0) = -1$, $y'(0) = 2$. Give the open connected interval. (4marks)

4-a) State and prove the theorem of dependence of solutions of the I.V problem , $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$. ,upon slight change in the initial conditions . (12marks)

b) For the initial value problem $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$. Prove that their exist at least one solution iff :

$\varphi(x) = y_0(x) + \int_{x_0}^{x} f(x, \varphi(x))\, dx$. (8marks)

دور: يناير 2016 الزمن : ساعتان التاريخ : / 1 / 2016	كلية العلوم – قسم الرياضيات	المستوى: الرابع . المادة : هندسة تفاضلية كود المادة : ر416 البرنامج : رياضيات

Answer the following questions: *Full mark : 80*

1-a) Define : the regular curve , the arc- length function . Show that the curve $\alpha(t) = (e^t \cos t, e^t \sin t)$ is regular and find the speed and the arc- length function based at $t = 0$.

b) Deduce the expression of the second fundamental form II of a regular surface and prove that $II = 0$ for a plane .

2- a) Find Frenet formulae for a space curve $\alpha(t)$ and show that

$$\frac{d^2\alpha}{dt^2} = \frac{d^2 s}{dt^2} T + K\left(\frac{ds}{dt}\right)^2 N \text{ and find } \frac{d^3\alpha}{dt^3}$$

b) Define the Gaussian curvature K and the normal curvature K_n of a regular surface . And find K for the torus

$$r(u,v) = ((a+b\cos u)\cos v, (a+b\cos u)\sin v, b\sin u)$$

3- a) Define T , N , B , the torsion τ of a space curve $\alpha(t)$ and prove that

$$\tau = \frac{\dot\alpha \times \ddot\alpha \cdot \dddot\alpha}{|\dot\alpha \times \ddot\alpha|^2} .$$

b) Prove that the surface

$$r(u,v) = (f(u)\cos v, f(u)\sin v, h(u))$$

is regular and find the first fundamental form I and the second fundamental form II for it .

4- a) Define the unit normal surface N^* and the tangent plane $T_p(S)$ for a regular surface S . Find N^* and the equation of $T_p(S)$ for the surface $z = x^2 - y^2$, $p = (1, 1, 0)$

b) Compute Frenet apparatus T , N , K , and the torsion τ for the curve $\alpha(t) = \left(\frac{t^2}{2}, \sin t, \cos t\right)$.

Best wishes