

Answer the following four questions:

1- 1- Give two distinct equivalent definitions of a lattice. (5 points)

And then give an example of each of:

(i) A partially ordered set but not a lattice, (3 points)

(ii) Find all posets with 4 elements and determine which one is a meet-lattice, a join-semilattice or lattice. (3 points)

(iii) A poset has more than one maximal element. (3 points)

(iv) A poset has only one maximal element. (3 points)

(v) All lattices of order 5. (3 points)

2- In a lattice $L(\vee, \wedge)$ prove that:

(i) There is always at least one maximal element in a finite poset. (5 points)

(ii) If ρ is a poset, then ρ^{-1} is also a poset. (5 points)

(iii) If $a_1 \leq b_1$ and $a_2 \leq b_2$, then $a_1 \vee a_2 \leq b_1 \vee b_2$. (5 points)

(iv) If f is a \vee -homomorphism, then f is a \leq -homomorphism. (5 points)

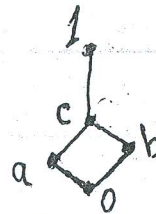
3- a- Give two equivalent definitions of ideals of a lattice (L, \vee, \wedge) . (10 points)

And prove the equivalence between them.

b- Define the principle ideal $a/0$ for $a \in L$,

and prove that $a/0$ is an ideal of L .

c- determine the lattice of ideals of the lattice



(5 points)

(5 points)

4- Prove that : The lattice $L = (L; \vee, \wedge)$ is not modular if and only

if L contains N_5 as a sublattice. And then give an example

of a not modular lattice of order 6

(20 points)

Full marks 80 points

دور يناير ٢٠١٦
الزمن: ساعتان
التاريخ: 5/1/2016



كلية العلوم - قسم الرياضيات

الفرقة: الرابعة
الشعبة: ر+ح ص
المادة: بحوث رياضية (٤٤)

Answer all questions:

Question[1]

a- Define:

- (i) The convex set (ii) The convex function (iii) The feasible region

b- Choose the correct answer:

1- Which are the non-negative constraints of a linear program?

- a. $X_1, X_2 > 0$ b. $X_1, X_2 \geq 0$ c. $X_1 + X_2 > 0$ d. $X_1 + X_2 \geq 0$

2- Equation $X_1 + 2X_2 = 5$ corresponds to a _____ in a two-dimensional coordinate system.

- a. point b. straight line c. circle d. parabola

3- A constraint must be an inequality (either " \leq " or " \geq "). A constraint cannot be an equation (" $=$ ").

- a. True b. False

4- A solution is feasible if it makes _____

- a. all the constraints hold. b. at least one constraint hold

c- Show that $f(x) = |x| \quad \forall x \in R$ is a convex function?

(الدرجة ٢٠)

Question[2]

a- True or false :

(i) The union of two convex sets is convex set.

(ii) If $f: R^n \rightarrow R$ be a concave function over a convex set S then $\frac{-1}{f(x)} \cdot f < 0$ is concave function.

(iii) The extreme points of the set $\{(x,y): |x| \leq 1, |y| \leq 1\}$ are $\{(1,-1), (1,1), (-1,1), (-1,-1)\}$

(iv) Minimize Z = Maximize $\{-Z\}$

(v) The dual of the dual is primal

(vi) If f_1 and f_2 are convex functions then $f_1 + f_2$ is also convex

b- By using the Simplex method solve the problem:

$$\begin{aligned} \text{Max } Z &= 2x_1 + 3x_2 && \text{Such that.} \\ 2x_1 + 3x_2 &\leq 1, && 8x_1 + 2x_2 \leq 3, && x_1, x_2 \geq 0 \end{aligned}$$

(اقلب الصفحة)

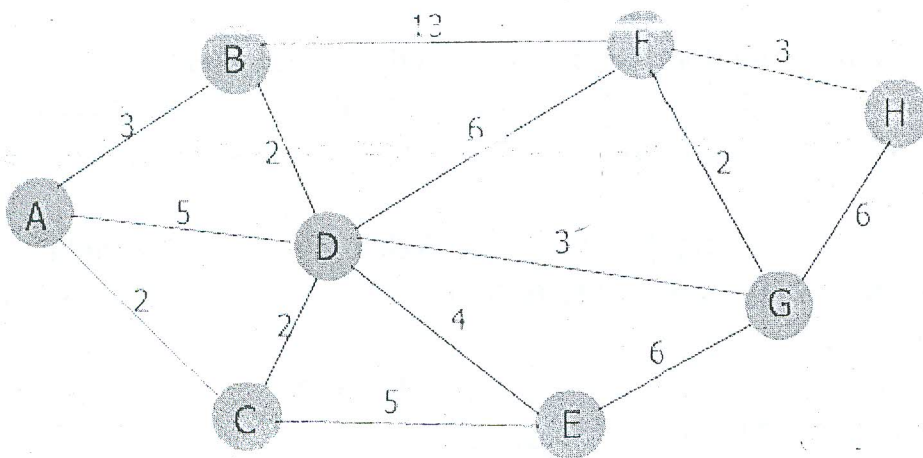
c- Solve the following transportation problem using the Vogel method:

	D1	D2	D3	D4	Availability
O1	6	4	1	5	14
O2	8	9	2	7	16
O3	4	3	6	2	5
Requirement	6	10	15	4	

(الدرجة ٣٠)

Question[3]

a- Solve the minimum- spanning problem for given network:



b- For the following linear program, find the dual and solve it graphically:

$$\text{Min } Z = 2x_1 + 3x_2 \quad \text{Such that. } 2x_1 + 3x_2 \geq 1, \quad 8x_1 + 2x_2 \geq 3, \quad x_1, x_2 \geq 0$$

(الدرجة ٣٠)

د. محمد عبد الرحمن

مع تمنياتي بالنجاح والتفوق



Final-term Exam
Stochastic Process
First Semester 2015 G
Time Allowed is 2 hours

Mansoura University
College of Science
Dept. of Math.

Answer the following Questions:

Q.1

a) Define the following concepts:

1. The Markov chain and give an example.
1. Recurrent State - Transient state - Absorbing state.
2. Irreducible Markov chain please give an example

b) Four coins are placed in a row on a table. At every stage, a coin is selected at random and turned over. Assume that X_n denotes the total number of heads out of four coins after the n -th trial.

I. Show that $\{X_n\}$ is a Markov chain.

II. Find the first step transition probability matrix P_{ij} of this Markov chain.

III. If we start with 3 heads ($P^0 = (0,0,1,0,0)$), find the probability that, there are one head after 1st trial.

c) Write the Chapman-Kolmogorov equations.

Q.2

a) A Markov chain $\{X_n\}$ with transition probability matrix

$$P = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ \frac{1}{4} & 0 & \frac{3}{4} & 0 & 0 \\ 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{3}{4} & 0 & \frac{1}{4} \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

If the initial distribution of the process is $P^{(0)} = (0,0,0,1,0)$

Find the marginal probabilities, $P(X_2 = 1)$, $P(X_2 = 3)$

b) A Markov chain with t.p.m.

$$\begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix}$$

where $0 < \alpha < 1$, $0 < \beta < 1$

1. Find the eigen values of P .
2. Find the transition probability matrix of n step.
3. Find $\lim_{n \rightarrow \infty} P^{(n)}$.

Q.3

a. If T_1, T_2, \dots, T_n denote the time measured from zero at which events occur according to Poisson process with constant rate λ , then prove that:

$$F_{T_n}(t) = P(T_n \leq t) = 1 - \sum_{r=0}^{n-1} \frac{(\lambda t)^r}{r!} e^{-\lambda t}$$

Where T_n is the time till the n -th event

b. Telephone calls arrive at an exchange according to a Poisson process at the rate 2 calls per minute. If a telephone calls just received, find the probability that the next three calls will be arrive after 2 minutes.

من فضلك يا قلب اورت

Q.4

a. Assume that $P_n(t) = P\{X(t) = n | X(0) = 0\}$ and $X(t)$ is the number of times an event occurs in the time interval $(0, t)$. Suppose that the event occurs under the following conditions:

1. Births which occur in disjoint intervals are independent of each other.

2. When there n members in the population at time t

• $P\{\text{one event in } (t, t + \delta t)\} = \lambda_n \delta t + o(\delta t),$

• $P\{\text{two or more events in } (t, t + \delta t)\} = o(\delta t),$

Let $P_n(t)$ denote the probability that the population size at time t is n , prove that

$$\frac{dP_n(t)}{dt} + \lambda_n P_n(t) = \lambda_{n-1} P_{n-1}(t), \quad n \geq 1$$

$$\frac{dP_0(t)}{dt} = -\lambda P_0(t)$$

b. For linear birth process $\lambda_n = n\lambda$, and the condition

$$P_n(0) = \begin{cases} 1 & \text{if } n = 1 \\ 0 & \text{if } n \neq 1 \end{cases}$$

prove that

$$P_n(t) = e^{-\lambda t} (1 - e^{-\lambda t})^{n-1}, \quad n \geq 1$$

where $P_n(t) = P\{X(t) = n | X(0) = 0\}$.

مع اطيب التمنيات بالنجاح أ.د. عوض الجوهرى

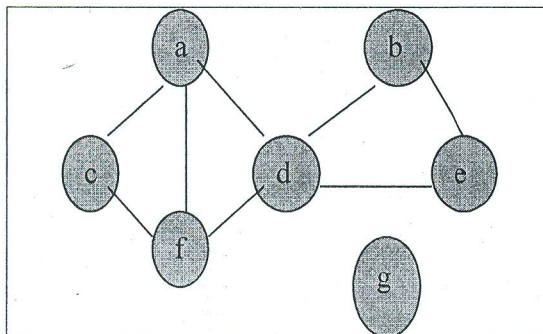


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Answer the Following questions

1. **Big-O Analysis, Introductuin to ADT, and Graphs** [20 points]

- a) What are **Best-Case**, and **Average-Case** complexity of Binary Search? [5 points]
 And what are **Best-Case**, **Worst-Case**, **Average-Case** complexity of Linear Search?
- b) Using Data Structures, write an ADT [5 points]
 For **Course** (Characteristics and Operations).
 Any course has a list of **students** and a **teacher**.
 We can perform operations such as: **adding students**, **assigning a teacher**, **getting the number of students**, **getting the teacher**, **removing a student**.
- c) Represent the following graph as **adjacency matrix** and **incident matrix** [10 points]



2. **Abstract data types (ADTs)** [40 points]

- a) Trace the following program by drawing the **Linked List** structure [20 points]

```
List<int> list=new List<int>();
list.Append (25);
list.Append (79);
list.Append (120);
list.Insert (100);
```

- b) Evaluate the following **postfix** expression using **Stack ADT** [5 points]

1 3 8 2 / + * 2 ^ 3 +

Note that ^ is the power, so (7 ^ 2) in infix = (7 2 ^) in postfix = 49

- c) List **inorder**, **preorder**, **postorder** traversal on the Binary tree [15 points]

```
BinaryTree tree;
tree.add(5);
tree.add(3);
tree.add(8);
tree.add(10);
tree.add(9);
```



Mansoura University, Faculty of Science, Mathematics Department

امتحان مقرر تحليل عددي (2) - ر 413 لطلاب السنة الرابعة بكلية العلوم شعبتي الإحصاء وعلوم الحاسب + الرياضيات
الفصل الدراسي الأول يناير 2016

Time Allowed: 2 hours

Answer the following questions . All questions carry equal marks . مسموح باستخدام الآلة الحاسبة

Question (1)

- 1-a) Describe the Butcher's table for the 4th order Runge-Kutta (RK4) method that uses 4 stages per step.
1-b) Express the Chebyshev polynomial of the first kind , $T_n(x)$, $n=1,2,\dots$ as a tri-diagonal determinant.
How many real zeros in the open interval $] -1,1[$ for $T_n(x)$? . Find these zeros.
1-c) Apply the Adams-Moulton method to approximate y at $x=0.4$ given that $\frac{dy}{dx} = y^2 + 4$, $y(0) = 0$.

Question (2)

- 2-a) Find the root between 0 and 1 of the equation $f(x) = x^3 - 6x + 4 = 0$ correct to four decimal places.
2-b) In some determination of the volume v of carbondioxide dissolved in a given volume of water at temperature θ , the following pairs of values were obtained

v	0	5	10	15
θ	1.8	1.45	1.8	1.00

Obtain by the method of least-squares a relation of the form $v = a + b \theta$.

Question (3)

- 3-a) Use the RK4 method to approximate y and z when $x=0.1$ for that particular solution of the system:

$$\begin{cases} \frac{dy}{dx} = z = f(x, y, z) \\ \frac{dz}{dx} = 4y - 2xz = g(x, y, z) \end{cases}$$

satisfying $y = 0.2$, $z = 0.5$ when $x = 0$.

- 3-b) Find a quadratic least-squares approximation of the form : $p_2(x) = a_0 T_0(x) + a_1 T_1(x) + a_2 T_2(x)$

For the function $f(x) = \frac{1}{(1+x)^2}$, $-1 \leq x \leq 1$. Compute $\left| p_2(0) - f(0) \right|$.

Kind regards

Prof. Dr. Moawwad El-Mikkawy



Fourth Level: Statistics and computer science program

Answer the following questions:

First Question: [20 Marks]

a) Design a learning algorithm to solve a classification problem approximately linear separable. [7 Marks]

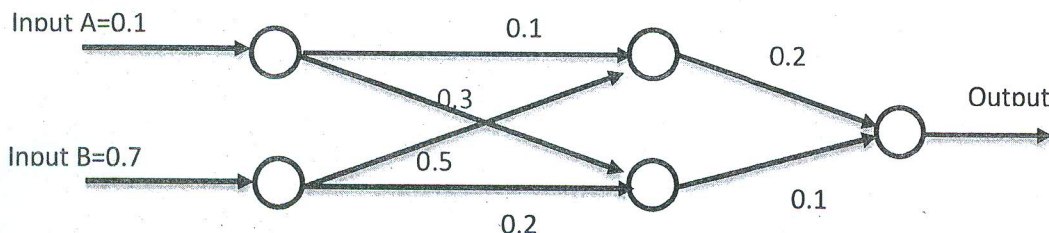
b) Consider the following classification problem $\left\{ p_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, t_1 = -1 \right\}$,

$\left\{ p_2 = \begin{bmatrix} 0 \\ -1 \\ -1 \end{bmatrix}, t_2 = 1 \right\}, \left\{ p_3 = \begin{bmatrix} -1 \\ -0.5 \\ -1 \end{bmatrix}, t_3 = 1 \right\}$, Train a perceptron network to solve this problem using the perceptron learning rule (Take $\eta=0.1$ and $W_0=(1,-1,0)$). [13 Marks]

Second Question: [20 Marks]

a) Suppose we wish to store three vectors $i_1=(1,-1,-1,1,1)$, $i_2=(-1,1,-1,1,-1)$, and $i_3=(1,-1,1,-1,1)$. Design a Hamming network to find the winning neuron when the input vector $(1,1,1,-1,-1)$ is presented to this network. [7 Marks]

b) Consider the simple network below:



Assume that the neurons have a sigmoid activation function:

1. Perform a forward pass on the network.
2. Perform a reverse pass (training) once (desired=1 and learning rate=1)

[13 Marks]

Third Question: [20 Marks]

a) Draw the structure of SOM network and write its learning algorithm.

[8 Marks]

b) Apply LVQ algorithm to the following data: $i_1=(1.1,1.7,1.8)$, $i_2=(0,0,0)$, $i_3=(0,0.5,1.5)$, $i_4=(1,0,0)$, $i_5=(0.5,0.5,0.5)$, $i_6=(1,1,1)$. Start by $\eta(t)=0.5$ and

$$W(0) = \begin{pmatrix} w_1: 0.2 & 0.7 & 0.3 \\ w_2: 0.1 & 0.1 & 0.9 \\ w_3: 1 & 1 & 1 \end{pmatrix}. \text{ (One epoch).}$$

[12 Marks]

End of the Exam, Good Luck
 Examiner: Dr. Yasser Fouda

Mansoura University
Faculty of Science
Department of
Mathematics



First- term 2015/2016

Course: Statistical Theory (2)
Math 431

Time: 2 hours

Final term- exam

Program: Statistics &
Computer Sciences
Level 4

Full marks : 80

أجب على الأسئلة التالية:

السؤال الأول (20 درجة)

يرغب طبيب في معرفة أي العقارين A ، B أكثر فعالية في تقليل متوسط زمن الشفاء من مرض ما. فقام بتقسيم عدد من المرضى الى مجموعتين بطريقة عشوائية، المجموعة الأولى وعددها 10 تلقت العقار A ، والمجموعة الثانية وعددها 9 تلقت العقار B. وحصل على النتائج التالية:

$$\bar{X}_A = 10 \text{ days}, S_A^2 = 10 (\text{days})^2, \bar{X}_B = 9 \text{ days}, S_B^2 = 9 (\text{days})^2$$

وبفرض أن زمن الشفاء من هذا المرض يخضع للتوزيع الطبيعي، اذن

(أ) اختبر تجانس تبايني زمن الشفاء للعقارين $\alpha = 0.05$

(ب) اختبر الفرض القائل أن فعالية العقار A أقل أو تساوى فعالية العقار B. أعتبر $\alpha = 0.01$

السؤال الثاني (25 درجة)

(أ) في عينة مكونة من 100 طالب وطالبة كانت أعداد الناجحين والراسبين من الجنسين في مقرر الإحصاء كالتالي:

الجنس	النتيجة	ناجح	راسب
طالب	40	15	
طالبة	35	10	

اختبر ما اذا كانت نتائج الامتحان مستقلة عن الجنس أم لا. $\alpha = 0.05$

(ب) الجدول التالي يبين أعداد الطلاب الناجحين والراسبين في مقرر للإحصاء من ثلاث شعب A, B, C ، حيث قام بالتصحيح ثلاثة أساتذة مختلفين، كل منهم صحح شعبة منفردا.

حالة الطالب		شعب المقرر والمصححون
ناجح	راسب	
50	5	الشعبة A والمصحح I
47	14	الشعبة B والمصحح II
56	8	الشعبة C والمصحح III

اختبر ما اذا كانت نسب النجاح والرسوب للمصححين الثلاثة للشعب الثلاثة متجانسة. $\alpha = 0.01$

السؤال الثالث (20 درجة)

في دراسة لوقت انتظار عمال مؤسسة ما للحافلة التي يستخدمونها في الذهاب الى

أنظر خلف الورقة

العمل، سجلت فترات انتظارهم للحافلة بالدقائق لعينة من 15 يوما وكانت النتائج كالتالي:

4, 8, 5, 7, 2, 8, 5, 9, 6, 1, 5, 6, 5, 9, 5.

استخدم اختبار الإشارة وكذلك اختبار الرتب لولكوسون لاختبار إذا كان متوسط زمن انتظارهم للحافلة أكبر من 5 دقائق. هل تلاحظ فرقا بين الاختبارين؟ $\alpha = 0.05$.

السؤال الرابع (15 درجات)

يعتقد البعض أن متوسط الأطوال لطلاب الجامعة أكبر منه لدى الطالبات. تم اختيار عينة عشوائية من الطلاب عددها 49 طالبا، فكان متوسط أطوالهم 170 سم وبانحراف معياري 33 سم، كما اختيرت عينة عشوائية من الطالبات عددها 36 طالبة، فكان متوسط أطوالهن 168 سم وبانحراف معياري 30 سم. فهل هذا الاعتقاد صحيح؟ $\alpha = 0.05$

$$F(9,8;0.975) = 4.36, F(9,8;0.025) = 0.244 \text{ و } F(8,9;0.975) = 4.10,$$

$$F(8,9;0.025) = 0.23, \quad t(0.99;17) = 2.567, t(0.995;17) = 2.898,$$

$$t(0.975;20) = 2.086, \chi^2_{2,0.01} = 9.21, \chi^2_{1,0.05} = 3.841, Z_{0.95} = 1.645$$

My best wishes...

Prof. Ahmed H. El-Bassiouny