Existence of heteroclinic and homoclinic orbits in two different chaotic dynamical systems

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Abstract
This paper presents the existence of Si'lnikov orbits in two different chaotic systems belong to the class of Lorenz systems, more exactly in the Lu system and in the Zhou's system. Both systems have exactly two heteroclinic orbits which are symmetrical with respect to the z-axis by using the undetermined coefficient method. The existence of the homoclinic orbit for the Zhou's system has been proven also by using the undetermined coefficient method. As a result, the Si'lnikov criterion along with some technical conditions guarantees that Lu and Zhou's systems have both Smale horseshoes and horseshoe type of chaos. Moreover, the geometric structures of attractors are determined by these heteroclinic orbits. (C) 2012 Elsevier Inc. All rights reserved.

KeyWords: Si'lnikov criterion; Lu system; Zhou's system; Heteroclinic orbits; Homoclinic orbits; Smale horseshoes; Undetermined coefficients method
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Global Attractivity and Periodic Character of Difference Equation of Order Four

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Abstract
We investigate the global convergence result, boundedness, and periodicity of solutions of the recursive sequence x(n+1) = ax(n) + ((bx(n-1) + cx(n-2) + dx(n-3))/(alpha x(n-1) + beta x(n-2) + gamma x(n-3))), n = 0, 1, . . ., where the parameters a, b, c, d, alpha, beta, and gamma are positive real numbers and the initial conditions x(-3), x(-2), x(-1), and x(0) are positive real numbers.
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The Form of the Solutions and Periodicity of Some Systems of Difference Equations

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Abstract
This paper is devoted to get the form of the solutions and the periodic nature of the following systems of rational difference equations x(n+1) = x(n-5)/(-1 + xn-5yn-2), y(n+1) = y(n-5)/(+/- 1 +/- y(n-5)x(n-2)), where the initial conditions are real numbers.
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Adaptive Feedback Control for Chaos Control and Synchronization for New Chaotic Dynamical System

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Abstract
This paper investigates the problem of chaos control and synchronization for new chaotic dynamical system and proposes a simple adaptive feedback control method for chaos control and synchronization under a reasonable assumption. In comparison with previous methods, the present control technique is simple both in the form of the controller and its application. Based on Lyapunov's stability theory, adaptive control law is derived such that the trajectory of the new system with unknown parameters is globally stabilized to the origin. In addition, an adaptive control approach is proposed to make the states of two identical systems with unknown parameters asymptotically synchronized. Numerical simulations are shown to verify the analytical results.

KeyWords: SECURE COMMUNICATION; ACTIVE CONTROL; HYPERCHAOTIC SYSTEMS; LORENZ SYSTEMS

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Adaptive Modified Function Projective Synchronization between Two Different Hyperchaotic Dynamical Systems

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Abstract
This work investigates modified function projective synchronization between two different hyperchaotic dynamical systems, namely, hyperchaotic Lorenz system and hyperchaotic Chen system with fully unknown parameters. Based on Lyapunov stability theory, the adaptive control law and the parameter update law are derived to achieve modified function projective synchronized between two diffierent hyperchaotic dynamical systems. Numerical simulations are presented to demonstrate the effectiveness of the proposed adaptive controllers.

KeyWords: CHAOTIC SYSTEMS; LORENZ SYSTEM; GENERALIZED SYNCHRONIZATION; UNCERTAIN PARAMETERS; LAG SYNCHRONIZATION; CHEN SYSTEM; OSCILLATORS; FEEDBACK; ROSSLER; PHASE

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