

# On chaos control and synchronization of the commensurate fractional order Liu system

**Author(s):**

Matouk, AE (Matouk, A. E.)<sup>[1,2]</sup>

**E-mail:** aematouk@hotmail.com

[ 1 ] Hail Univ, Fac Sci, Dept Math, Hail 2440, Saudi Arabia

Hegazi, AS (Hegazi, A. S.)<sup>[2]</sup>, Ahmed, E (Ahmed, E.)<sup>[2]</sup>

**E-mail:** hegazi@mans.edu.eg; magd45@yahoo.com

[ 2 ] Mansoura Univ, Dept Math, Fac Sci, Mansoura 35516, Egypt

## Abstract:

In this work, we study chaos control and synchronization of the commensurate fractional order Liu system. Based on the stability theory of fractional order systems, the conditions of local stability of nonlinear three-dimensional commensurate fractional order systems are discussed. The existence and uniqueness of solutions for a class of commensurate fractional order Liu systems are investigated. We also obtain the necessary condition for the existence of chaotic attractors in the commensurate fractional order Liu system. The effect of fractional order on chaos control of this system is revealed by showing that the commensurate fractional order Liu system is controllable just in the fractional order case when using a specific choice of controllers. Moreover, we achieve chaos synchronization between the commensurate fractional order Liu system and its integer order counterpart via function projective synchronization. Numerical simulations are used to verify the analytical results. (C) 2012 Elsevier B.V. All rights reserved.

**KeyWords:** Commensurate fractional order Liu system; Stability conditions; Chaos; Chaos control, Projective synchronization

**Published in :** COMMUNICATIONS IN NONLINEAR SCIENCE AND NUMERICAL SIMULATION Volume: 18 Issue: 5 Pages: 1193-1202 DOI: 0.1016/j.cnsns.2012.09.026

Published: MAY 2013

## References:

- [1] O. Heaviside
- Electromagnetic Theory
- Chelsea, New York (1971)
- [2] R.L. Bagley, R.A. Calico
- Fractional order state equations for the control of viscoelastically damped structures
- J. Guid. Control Dyn., 14 (1991), pp. 304–311
- [3] A.M.A. El-Sayed
- Fractional-order diffusion-wave equation
- Int. J. Theor. Phys., 35 (1996), pp. 311–322
- [4] A.E.M. El-Misiery, E. Ahmed
- On a fractional model for earthquakes
- Appl. Math. Comput., 178 (2006), pp. 207–211
- [5] R. Hilfer (Ed.), Applications of Fractional Calculus in Physics, World Scientific, New Jersey (2000)

- [6]
  - I. Podlubny
  - Fractional Differential Equations
  - Academic Press, New York (1999)
- [7]
  - E. Ahmed, A.S. Elgazzar
  - On fractional order differential equations model for nonlocal epidemics
  - Phys. A, 379 (2007), pp. 607–614
- [8]
  - A.M.A. El-Sayed, A.E.M. El-Mesiry, H.A.A. El-Saka
  - On the fractional-order logistic equation
  - Appl. Math. Lett., 20 (2007), pp. 817–823
- [9]
  - E. Ahmed, A.M.A. El-Sayed, H.A.A. El-Saka
  - Equilibrium points, stability and numerical solutions of fractional-order predator-prey and rabies models
  - J. Math. Anal. Appl., 325 (2007), pp. 542–553
- [10]
  - N. Laskin
  - Fractional market dynamics
  - Phys. A, 287 (2000), pp. 482–492
- [11]
  - B. Xin, T. Chen, Y. Liu
  - Projective synchronization of chaotic fractional-order energy resources demand-supply systems via linear control
  - Commun. Nonlinear Sci. Numer. Simulat., 16 (2011), pp. 4479–4486
- [12]
  - W.M. Ahmad, R. El-Khazali
  - Fractional-order dynamical models of love
  - Chaos Solitons Fract., 33 (2007), pp. 1367–1375
- [13]
  - L. Song, S. Xu, J. Yang
  - Dynamical models of happiness with fractional order
  - Commun. Nonlinear Sci. Numer. Simulat., 15 (2010), pp. 616–628
- [14]
  - M. Caputo
  - Linear models of dissipation whose Q is almost frequency independent-II
  - Geophys. J. R. Astron. Soc., 13 (1967), pp. 529–539
- [15]
  - E.N. Lorenz
  - Deterministic non-periodic flows
  - J. Atmos. Sci., 20 (1963), pp. 130–141
- [16]
  - C. Liu, T. Liu, L. Liu, K. Liu
  - A new chaotic attractor
  - Chaos Solitons Fract., 22 (2004), pp. 1031–1038
- [17]
  - X.Y. Wang, M.J. Wang
  - A chaotic secure communication scheme based on observer
  - Commun. Nonlinear Sci. Numer. Simulat., 14 (2009), pp. 1502–1508

- [18] A.E. Matouk
  - Dynamical analysis, feedback control and synchronization of Liu dynamical system
  - Nonlinear Anal. Theory, 69 (2008), pp. 3213–3224
  - [19] I. Grigorenko, E. Grigorenko
  - Chaotic dynamics of the fractional Lorenz system
  - Phys. Rev. Lett., 91 (2003), p. 034101
  - [21] C.P. Li, G.J. Peng
  - Chaos in Chen's system with a fractional order
  - Chaos Solitons Fract., 22 (2004), pp. 443–450
  - [22] C.G. Li, G. Chen
  - Chaos and hyperchaos in the fractional-order Rössler equations
  - Phys. A, 341 (2004), pp. 55–61
- [23] M. Shahiri, R. Ghaderi, N. Ranjbar, S. Hosseinnia, S. Momani
- Chaotic fractional-order Coul- let system: synchronization and control approach
  - Commun. Nonlinear Sci. Numer. Simulat., 15 (2010), pp. 665–674
  - [24] A.E. Matouk
  - Chaos, feedback control and synchronization of a fractional-order modified autonomous Van der Pol-Duffing circuit
  - Commun. Nonlinear Sci. Numer. Simulat., 16 (2011), pp. 975–986
  - [25] X.Y. Wang, M.J. Wang
  - Dynamic analysis of the fractional-order Liu system and its synchronization
  - Chaos, 17 (2007), p. 033106
  - [26] L. Jun-Jie, L. Chong-Xin
  - Realization of fractional-order Liu chaotic system by circuit
  - Chin. Phys., 16 (2007), pp. 1586–1590
  - [27] S. Bhalekar, V. Daftardar-Gejji
  - Fractional ordered Liu system with time-delay
  - Commun. Nonlinear Sci. Numer. Simulat., 15 (2010), pp. 2178–2191
  - [28] W.M. Ahmad, A.M. Harb
  - On nonlinear control design for autonomous chaotic systems of integer and fractional orders
  - Chaos Solitons Fract., 18 (2003), pp. 693–701
  - [29] C.G. Li, X.F. Liao, J.B. Yu
  - Synchronization of fractional order chaotic systems
  - Phys. Rev. E, 68 (2003), p. 067203
- [30] M.S. Tavazoei, M. Haeri

- Synchronization of chaotic fractional-order systems via active sliding mode controller
- Phys. A, 387 (2008), pp. 57–70  
[\[31\]](#)
- G.H. Erjaee, S. Momani
- Phase synchronization in fractional differential chaotic systems
- Phys. Lett. A, 372 (2008), pp. 2350–2354  
[\[32\]](#)
- A.E. Matouk
- Chaos synchronization between two different fractional systems of Lorenz family
- Math. Prob. Eng. (2009) 2009 [Article ID 572724]  
[\[33\]](#)
- A.E. Matouk
- Stability conditions, hyperchaos and control in a novel fractional order hyperchaotic system
- Phys. Lett. A, 373 (2009), pp. 2166–2173  
[\[34\]](#)
- Z. Odibat, N. Corson, M. Aziz-Alaoui, C. Bertelle
- Synchronization of chaotic fractional-order systems via linear control
- Int. J. Bifurcat. Chaos, 20 (2010), pp. 81–97  
[\[35\]](#)
- S. Bhalekar, V. Daftardar-Gejji
- Synchronization of different fractional order chaotic systems using active control
- Commun. Nonlinear Sci. Numer. Simulat., 15 (2010), pp. 3536–3546  
[\[36\]](#)
- P. Zhou, W. Zhu
- Function projective synchronization for fractional-order chaotic systems
- Nonlinear Anal. Real, 12 (2011), pp. 811–816  
[\[37\]](#)
- M.M. Asheghian, M.T.H. Beheshti, M.S. Tavazoei
- Robust synchronization of perturbed Chen's fractional-order chaotic systems
- Commun. Nonlinear Sci. Numer. Simulat., 16 (2011), pp. 1044–1051  
[\[38\]](#)
- L. Pan, W. Zhou, L. Zhou, K. Sun
- Chaos synchronization between two different fractional- order hyperchaotic systems
- Commun. Nonlinear Sci. Numer. Simulat., 16 (2011), pp. 2628–2640  
[\[39\]](#)
- H. Taghvafard, G.H. Erjaee
- Phase and anti-phase synchronization of fractional order chaotic systems via active control
- Commun. Nonlinear Sci. Numer. Simulat., 16 (2011), pp. 4079–4088  
[\[40\]](#)
- A.S. Hegazi, A.E. Matouk
- Dynamical behaviors and synchronization in the fractional order hyperchaotic Chen system
- Appl. Math. Lett., 24 (2011), pp. 1938–1944  
[\[41\]](#)
- K. Diethelm, N.J. Ford
- Analysis of fractional differential equations

- J. Math. Anal. Appl., 265 (2002), pp. 229–248
- [\[42\]](#)
- Matignon D. Stability results for fractional differential equations with applications to control processing, in: Computational engineering in systems and application multi-conference, vol. 2, IMACS, IEEE-SMC Proceedings, Lille, France; July 1996, pp. 963–8.
- [\[43\]](#)
- E. Ahmed, A.M.A. El-Sayed, H.A.A. El-Saka
- On some Routh-Hurwitz conditions for fractional order differential equations and their applications in Lorenz, Rössler, Chua and Chen systems
- Phys. Lett. A, 358 (2006), pp. 1–4
- [\[44\]](#)
- K. Diethelm, N.J. Ford, A.D. Freed
- A predictor-corrector approach for the numerical solution of fractional differential equations
- Nonlinear Dyn., 29 (2002), pp. 3–22
- [\[45\]](#)
- M.S. Tavazoei, M. Haeri
- A necessary condition for double scroll attractor existence in fractional-order systems
- Phys. Lett. A, 367 (2007), pp. 102–113
- [\[46\]](#)
- L.O. Chua, M. Komuro, T. Matsumoto
- The double-scroll family
- IEEE Trans. Circ. Syst., 33 (1986), pp. 1072–1118