New conditional integrable cases of motion of a rigid body with Kovalevskaya's configuration

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Abstract:
We consider the general problem of motion of a rigid body about a fixed point under the action of an axisymmetric combination of potential and gyroscopic forces. We introduce two new cases of this problem which are integrable on the zero level of the cyclic integral. The new cases are combined generalizations of several previously known ones, namely those of Kovalevskaya, Yehia, Sokolov, Yehia and Bedweihi and Goriatchev, by the introduction of additional parameters to the structure of each.

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Further classification of 2D integrable mechanical systems with quadratic invariants

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Abstract:
Four new integrable classes of mechanical systems on Riemannian 2D manifolds admitting a complementary quadratic invariant are introduced. Those systems have quite rich structure. They involve 11-12 arbitrary parameters that determine the metric of the configuration space and forces with scalar and vector potentials. Interpretations of special versions of them are pointed out as problems of motions of rigid body in a liquid or under action of potential and gyroscopic forces and as motions of a particle on the plane, sphere, ellipsoid, pseudo-sphere and other surfaces.

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New integrable systems with a quartic integral and new generalizations of Kovalevskaya's and Goriatchev's cases

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Abstract:
In his paper [1], one of us has introduced a method for constructing integrable conservative two-dimensional mechanical systems, on Riemannian 2D spaces, whose second integral is a polynomial in the velocities. This method was applied successfully in [2] to construction of systems admitting a cubic integral and in [3, 4] and [5] to cases of a quartic integral. The present work is devoted to construction of new integrable systems with a quartic integral. The potential is assumed to have the structure

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