

An efficient and simple refined theory for bending and vibration of functionally graded plates

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Abstract:

A two-dimensional theory of functionally graded plates is presented using a mixed variational approach. The theory accounts for a displacements field in which the in-plane displacements vary linearly through the plate thickness, while the out-of-plane displacement is a second-degree function of thickness coordinate. The advantages of the present theory are that it contains both the transverse normal strain and stress in complete consistence with the boundary conditions at the top and bottom surfaces of the plates without loss of its simplicity. Therefore, the rationale for the shear correction factor used in such theories is obviated. The bending and free vibration problems of isotropic plates with material properties varying in the thickness direction are solved. Numerical results for frequencies are presented for two-phase graded material with a power-law through the plate thickness variation of the volume fractions of the constituents based on Mori-Tanaka scheme. In addition, numerical results of transverse deflections are obtained for FG simply supported isotropic plates with Young's modulus varying exponentially through the thickness and constant Poisson's ratio. The validity of the present theory is investigated by comparing some of the present results with their counterparts obtained due to three-dimensional approaches by Qian et al. and by Kashtalyan. The influence of the transverse normal strain on the bending and vibration of the FG plates is illustrated. (C) 2009 Elsevier Ltd. All rights reserved..

KeyWords: Functionally graded plates; Two-dimensional formulation; Bending and vibration

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A refined zigzag nonlinear first-order shear deformation theory of composite laminated plates

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Abstract:

A refined nonlinear zigzag shear deformation theory of composite laminated plates is presented using a modified mixed variational formulation. The theory accounts for continuous piecewise layer-by-layer linear variation approximation in the thickness direction for the displacements. Moreover, it includes piecewise stress distributions satisfying the continuity conditions at the layer interfaces and the surface conditions. The advantages of this theory are that it recovers the interlaminar stresses, and does not need any shear correction factor used in other first-order theories. To assess this theory, the bending problems of symmetric and antisymmetric cross-ply laminated plates are solved. Some numerical results for the deflection and stresses are compared with their counterparts in the literature obtained due to three-dimensional elasticity solution and higher-order laminate theories. It is found that the present theory predicts the local and global responses of the laminated plates with excellent accuracy. (c) 2006 Elsevier Ltd. All rights reserved.

KeyWords: zigzag piecewise displacements; continuity conditions for the shear stresses; first-order theory; composite laminated plates; mixed variational formulation

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Non-linear design and control optimization of composite laminated plates with buckling and postbuckling objectives

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Abstract:

Multiobjective design and control optimization of composite laminated plates is presented to minimize the postbuckling dynamic response and maximize the buckling load. The control objective aims at dissipating the postbuckling elastic energy of the laminate with the minimum possible expenditure of control energy using a closed-loop distributed force. The layer thicknesses and fiber orientations are taken as design variables. The objectives of the optimization problem are formulated based on a shear deformation theory including the von-Karman non-linear effect for various cases of boundary conditions. The non-linear control problem is solved iteratively until an appropriate convergence criterion is satisfied based on Liapunov-Bellman theory. Liapunov function is taken as a sum of positive definite functions with different degrees. Comparative examples for three-layer symmetric and four-layer antisymmetric laminates are given for various cases of edges conditions. Graphical study is carried out to assess the accuracy of results obtained due to the successive iterations. The influences of the boundary conditions, orthotropy ratio, shear deformation, aspect ratio on the laminate optimal design are elucidated. (C) 2006 Elsevier Ltd. All rights reserved

KeyWords: structural design and control; composite laminates; buckling and postbuckling responses; non-linear analysis; shear deformation theory; various boundary conditions

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**THE MOTION OF A SYMMETRIC GYROSTAT AROUND THE CENTER
OF MASS IN A CIRCULAR ORBIT IN THE PRESENCE OF
VISCOELASTIC BARS AND DISC**

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Abstract:

In the present work we consider a symmetric gyrostator which has a homogeneous viscoelastic disc and two bars attached to it. Furthermore, the gyrostator has a rotor oriented inside it such that the rotor is statically and dynamically balanced. This system has a rotational motion around its center of mass in a circular orbit under a central gravitational field. Bending vibrations of the bars and the disc are accompanied by dissipation of energy, which is the cause of the evolution of the system's rotational motion. Using the method of separation of motion and averaging, the approximate equations describing the evolution of rotational motion in terms of Andoyer canonical variables are obtained. The stationary motions for the system are deduced, together with the conditions of its stability.

KeyWords: Attitude of satellites; viscoelastic bars and disc; stability of motion

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