

**Some inclusion relationships and integral-preserving properties of certain subclasses of  $p$ -valent meromorphic functions associated with a family of linear operator**

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**Abstract**

By making use of a general linear operator we introduce several new subclasses of multivalent meromorphic functions and investigate various inclusion relationships. Several interesting integral-preserving properties are also discussed.

**KeyWords:** Analytic function; Liu-Owa operator; Hadamard product; Differential subordination; Superordination.  $p$ -valent meromorphic functions; Hadamard product; linear operator; integral-preserving properties; inclusion relationships.

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# ARGUMENT ESTIMATES OF CERTAIN MEROMORPHICALLY P-VALENT FUNCTIONS ASSOCIATED WITH A FAMILY OF LINEAR OPERATOR

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## Abstract

Making use of linear operator we investigate some inclusion relationships and some argument properties of certain meromorphically p-valent functions. (C) 2011 Mathematical Institute Slovak Academy of Sciences.

**KeyWords:** meromorphically multivalent functions; Hadamard product; linear operator; argument estimates

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## ON CERTAIN CLASSES OF MEROMORPHICALLY $p$ -VALENT CONVEX FUNCTIONS

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### Abstract

Making use of a differential operator, which is defined here by means of the Hadamard product (or convolution), we introduce the class  $\Sigma(n)(p)(\alpha(1), \beta(1); \lambda)$  of meromorphically  $p$ -valent convex functions. The main object of this paper is to investigate various important properties and characteristics for this class. Further, a property preserving integrals is considered.

**KeyWords:**  $p$ -Valent; Hadamard product; Meromorphic; Convex; Jack's lemma

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### **On sandwich theorems for multivalent functions involving a generalized differential operator**

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#### **Abstract**

The purpose of this paper is to derive some subordination and superordination results for multivalent functions involving certain differential operator. (C) 2011 Elsevier Ltd. All rights reserved.

**KeyWords:** Analytic functions; Multivalent functions; Differential subordination; Differential superordination; Sandwich theorems; Generalized differential operator

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### **Inclusion properties of certain classes of meromorphic functions associated with the Wright generalized hypergeometric function**

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#### **Abstract**

The purpose of the present paper is to introduce several new classes of meromorphically univalent functions which are defined here by means of a linear operator. Also, we study various inclusion properties of these classes. (C) 2011 Elsevier Ltd. All rights reserved.

**KeyWords:** Meromorphic functions; Hadamard product; Differential subordination; Wright hypergeometric functions; Integral operator

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