Some Properties and Expressions of Solutions for a Class of Nonlinear Difference Equation

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Abstract
In this paper we study some properties of solutions of difference equation
$x_{n+1} = ax_{n-2} + bx_{n-2}x_{n-3}/cx_n + dx_{n-3}$, $n = 0, 1, \ldots$,
where the initial conditions x_{-3}, x_{-2}, x_{-1}, x_0 are arbitrary positive real numbers and a, b, c, d are constants. Also, we give the expressions of solutions for some special cases of this equation.

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Author(s): Elsayed, M.; El-Dessoky, Mohammed M.
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Author(s): Elsayed, E. M.
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Author(s): Elsayed, Elsayed Mohammed
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Adaptive Feedback Control for the Projective Synchronization of the Lu Dynamical System and its Application to Secure Communication

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Abstract

In this paper a simple adaptive linear feedback control method, based on the invariance principle of differential equations, is proposed for controlling the scaling factor to a desired value. We mathematically prove the synchronizability of the proposed simple adaptive modified projective synchronization control method. Also, we propose a communication scheme based on the adaptive modified projective synchronization of the new chaotic system. Based on the Lyapunov stability theory, an adaptive control law is proposed to make the states of two Lu attractors asymptotically synchronized. Numerical simulations are given to validate the proposed synchronization methods.

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Adaptive anti-synchronization of different chaotic dynamical systems

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Abstract
We have observed anti-synchronization phenomena in different chaotic dynamical systems. Anti-synchronization can be characterized by the vanishing of the sum of relevant variables. Anti-synchronization problem for different chaotic dynamical systems with fully unknown parameters in response system is analyzed. This technique is applied to achieve anti-synchronization between Lorenz system, Lu system and Four-scroll system. Numerical simulations are provided to verify the effectiveness of the proposed methods. (C) 2009 Elsevier Ltd. All rights reserved.
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Analysis of nonlinear triopoly game with heterogeneous players

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Abstract
A nonlinear triopoly game with heterogeneous players is presented. We consider three types of players: boundedly rational, adaptive, and naive. A triopoly game is modelled by a three dimensional discrete dynamical system. The stability conditions of the equilibrium points are analyzed. Numerical simulations are used to show bifurcation diagrams, phase portraits, sensitive dependence on initial conditions and fractal dimension. The chaotic behavior of the model has been stabilized on the Nash equilibrium point, by the use of the Pyragas delay feedback control method. (C) 2008 Elsevier Ltd. All rights reserved.
KeyWords: Triopoly game; Heterogeneous players; Fractal dimension; DFC method
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Chaotic dynamics of a discrete prey-predator model with Holling type II

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Abstract
A discrete-time prey-predator model with Holling type II is investigated. For this model, the existence and stability of three fixed points are analyzed. The bifurcation diagrams, phase portraits and Lyapunov exponents are obtained for different parameters of the model. The fractal dimension of a strange attractor of the model was also calculated. Numerical simulations show that the discrete model exhibits rich dynamics compared with the continuous model, which means that the present model is a chaotic, and complex one. (c) 2008 Published by Elsevier Ltd.

KeyWords: prey-predator model; Holling type II functional response; chaotic behavior; Layapunov exponents; fractal dimension
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