On Steiner Quasigroups of Cardinality 21

Author(s):
Armanious, MH (Armanious, M. H.)
E-mail: m.armanious@excite.com
Mansoura Univ, Dept Math, Fac Sci, Mansoura 35516, Egypt

Abstract

In [12] Quackenbush has expected that there should be subdirectly irreducible Steiner quasigroups (squags), whose proper homomorphic images are entropic (medial). The smallest interesting cardinality for such squags is 21. Using the tripling construction given in [1] we construct all possible nonsimple subdirectly irreducible squags of cardinality 21 (SQ(21)s). Consequently, we may say that there are 4 distinct classes of nonsimple SQ(21)s, based on the number n of sub-SQ(9)s for n = 0, 1, 3, 7. The squags of the first three classes for n = 0, 1, 3 are nonsimple subdirectly irreducible having exactly one proper homomorphic image isomorphic to the entropic SQ(3) (equivalently, having 3 disjoined sub-SQ(7)s). For n = 7, each squag SQ(21) of this class has 3 disjoint sub-SQ(7)s and 7 sub-SQ(9)s, we will see that this squag is isomorphic to the direct product SQ(7) x SQ(3). For n = 0, each squag SQ(21) of this class is a nonsimple subdirectly irreducible having three disjoint sub-SQ(7)s and no sub-SQ(9)s. In section 5, we describe an example for each of these classes Finally, we review all well-known classes of simple SQ(21)s.

Published in : ARS COMBINATORIA Volume: 104 Pages: 417-430 Published: APR 2012

References:

1-Title: Semi-planar Steiner Quasigroups of Cardinality 3n
Author(s): Armanious, M. H.
Source: Australisain Journal of Combinatorics Volume: 27 Pages: 13-21 Published: 2003
Times Cited: 2 (from All Databases)
2- Title: Subdirectly irreducible squags of cardinality 3n
Author(s): Armanious, MH; Tadros, SF; Dhshan, NM
Source: ARS COMBINATORIA Volume: 64 Pages: 199-210 Published: JUL 2002
3- Title: Subsquags and normal subsquags
Author(s): Armanious, MH
Source: ARS COMBINATORIA Volume: 59 Pages: 241-243 Published: APR 2001
4- Title: On semi-planar Steiner quasigroups
Author(s): Armanious, M. H.; Elbiomy, M. A.
Source: DISCRETE MATHEMATICS Volume: 309 Issue: 4 Pages: 686-692 DOI: 10.1016/j.disc.2007.12.097 Published: MAR 6 2009
5- Title: Semi-planar Steiner loops of cardinality 2n
Author(s): Armanious, MH
Source: DISCRETE MATHEMATICS Volume: 270 Issue: 1-3 Pages: 291-298 DOI: 10.1016/S0012-365X(03)00129-8 Published: AUG 28 2003
6- Title: [not available]
Author(s): Chein, O.; Pflugfelder, H.O.; Smith, J. D. H.
Source: Quasigroups and Loops, Theory and applications Volume: 8 Published: 1990
Publisher: Heldermann Verlag
7- Title: Sur la Structure de Certains Systems Triples de Steiner

Author(s): Doyen, J.
Source: Math. Z. Volume: 111 Pages: 289-300 Published: 1979
8- Title: Co-ordinatizing Steiner systems
Author(s): Ganter, B.; Werner, H.
Source: Ann. Discrete Math. Volume: 7 Pages: 3-24 Published: 1980
9- Title: [not available]
Author(s): Gratzer, G.
Source: Universal Algebra Published: 1997
Publisher: Springer Verlag, New York
10- Title: [not available]
Author(s): Haray, F.
Source: Graph Theory Published: 1969
11- Title: Steiner triple systems of order 19 and 21 with subsystems of order 7
Author(s): Kaski, Petteri; Ostergard, Patric R. J.; Topalova, Svetlana; et al.
Source: DISCRETE MATHEMATICS Volume: 308 Issue: 13 Pages: 2732-2741
DOI: 10.1016/j.disc.2006.06.038 Published: JUL 6 2008.

On semi-planar Steiner quasigroups

Author(s):
MH (Armanious, M. H.)[1]
E-mail: m.armanious@excite.com
Elbiomy, MA (Elbiomy, M. A.)[1]
Mansoura Univ, Dept Math, Fac Sci, Mansoura 35516, Egypt

Abstract

A Steiner triple system (briefly ST) is in 1-1 correspondence with a Steiner quasigroup or squag (briefly SQ) [B. Ganter, H. Werner, Co-ordinatizing Steiner systems, Ann. Discrete Math. 7 (1980) 3-24; C.C. Lindner, A. Rosa, Steiner quadruple systems: A Survey, Discrete Math. 21 (1979) 147-181]. It is well known that for each n equivalent to 1 or 3 (mod 6) there is a planar squag of cardinality n [J. Doyen, Sur la structure de certains systems triples de Steiner, Math. Z. 111 (1969) 289-300]. Quackenbush expected that there should also be semi-planar squags [R.W. Quackenbush, Varieties of Steiner loops and Steiner quasigroups, Canad. J. Math. 28 (1976) 1187-1198]. A simple squag is semi-planar if every triangle either generates the whole squag or the 9-element squag. The first author has constructed a semi-planar squag of cardinality 3n for all n > 3 and n equivalent to 1 or 3 (mod 6) [M.H. Armanious, Semi-planar Steiner quasigroups of cardinality 3n, Australas. J. Combin. 27 (2003) 13-27]. In fact, this construction supplies us with semi-planar squags having only nontrivial subsquags of cardinality 9. Our aim in this article is to give a recursive construction as n -> 3n for semi-planar squags. This construction permits us to construct semi-planar squags having nontrivial subsquags of cardinality >9. Consequently, we may say that there are semi-planar SQ(3(m) n)s (or semi-planar ST(3(m) n)s) for each positive integer m and each n equivalent to 1 or 3 (mod 6) with n > 3 having only medial subsquags at most of cardinality 3(nu) (sub-ST(3)(nu)) for each nu is an element of {1,2,...,m+ 1}.

Published in : DISCRETE MATHEMATICS Volume: 309 Issue: 4 Pages: 686-692 DOI: 10.1016/j.disc.2007.12.097 Published: MAR 6 2009

References:

- [1]
- M.H. Armanious

- Semi-planar Steiner quasigroups of cardinality $3n$
- Australas. J. Combin., 27 (2003), pp. 13–27
- [2]
- M.H. Armanious, S.F. Tadros, N.M. Dhshan
- Subdirectly irreducible squags of cardinality $3n$
- Ars Combin., 64 (2002), pp. 199–210
- [3]
- M.H. Armanious
- Subsquags and normal subsquags
- Ars Combin., 59 (2001), pp. 241–243
- [4]
- R.H. Bruck
- A Survey of Binary Systems
- Springer-Verlag, Berlin, Heidelberg, New York (1971)
- [5]
- J. Doyen
- Sur la structure de certains systems triples de Steiner
- Math. Z., 111 (1969), pp. 289–300
- [6]
- B. Ganter, H. Werner
- Co-ordinatizing Steiner systems
- Ann. Discrete Math., 7 (1980), pp. 3–24
- [7]
- G. Gratzer
- Universal Algebra
- (2nd ed.)Springer-Verlag, New York, Heidelberg, Berlin (1979)
- [8]
- A.J. Guelzow
- Representation of finite nilpotent squags
- Discrete Math., 154 (1996), pp. 63–76
- [9]
- M. Hall Jr.
- Automorphism of Steiner triple systems
- IBM J., 5 (1960), pp. 460–472
- [10]
- S. Klossek
- Kommutative Spiegelungsraume
- Mitt. Math. Sem. Univ. Giessen, 117 (1975)
- [11]
- C.C. Lindner, A. Rosa
- Steiner quadruple systems: A survey
- Discrete Math., 21 (1979), pp. 147–181
- [12]
- R.W. Quackenbush
- Varieties of Steiner loops and Steiner quasigroups
- Canad. J. Math., 28 (1976), pp. 1187–1198
- [13]
- R.W. Quackenbush
- Nilpotent block design I: Basic concepts for Steiner triple and quadruple systems
- J. Combin. Des., 7 (1999), pp. 157–171